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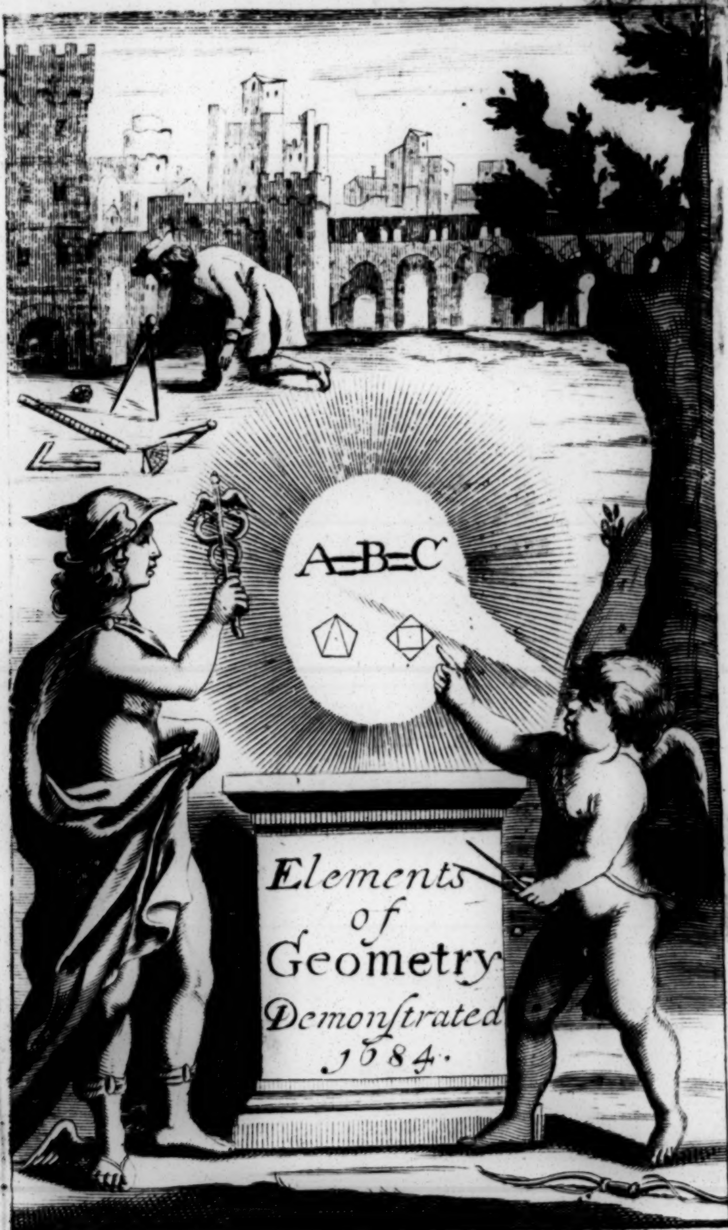

THE ELEMENTS OR PRINCIPLES OF GEOMETRIE.

London, Printed by *J. P.* for *Samuel Crowch* in *Cornhill*, *Richard Mount* on *Tower-Hill*, and *Awnsham Churchill*, at *Amen-Corner*. 1684

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TO THE

INGENUOUS and HOPEFUL

GENTLEMAN,

Mr. RALPH FREMAN,

SON OF

RALPH FREMAN,

OF *ASPEDEN* HALL
ESQUIRE;

THESE

ELEMENTS,

WHICH WERE AT FIRST
PREPARED FOR HIS USE,

ARE NOW WITH DUE RESPECT
PUBLICLY

DEDICATED.

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Advertiſement.

THE deſign of this *Treatiſe* being to reduce the *Elements* or *Principles Geometrie* into a ſhort Compaſs, for the benefit of thoſe that haſten to the Practical parts of this Studie, it ſeem'd fit to lay down the following *Definitions*, for the moſt part, as they uſually are expreſt, rather than ſpend time in giving reaſons for the contrary. Though we muſt confeſs with the Ingenuous *Borellio*, that the Angle of the Tangent (or a *mixt* Angle) is indeed no Angle. And perhaps *Geometrie* it ſelf may be better defin'd, the Art of meaſuring *Space*, rather than *Bodies*: And then, a *Surface*

 (being

(being the Boundary of such a Space) and Lines, the Boundaries of such Surface, as Points of such Lines, will all naturally appear to be immaterial things. Lastly, the Definition of *Reason* and *Aliquot parts* do not extend to incommensurable Quantities, which are omitted in this short Collection. And some Propositions about Proportion, according to the designed brevity, are rather illustrated than strictly demonstrated. Those that desire to be curious in these particulars can have recourse to larger Authors. This Summary was thought sufficient for an *Introduction*.

THE

INTRODUCTION.

FOR the assistance of those that first enter upon this Study, it may not be amiss to premise some few things by way of Introduction. First then, *Demonstration* is the highest degree of Proof that any matter admits of; fully satisfying the mind in the truth of what is undertaken, and leaving it no further room to doubt. And by consequence nothing is so apt to strengthen our Reason, to give us a clear notion of things, and secure us from being imposed upon by fallacies or shadows of proof, as conversing herein. And of all things that come under our Reason, there is nothing admits of so clear and evident Demonstration as *Geometry*. Where, by taking of matter to pieces we begin with the simplest parts (as they may be call'd) of *Body*, Lines and Points; and clearing our way before us, advance with no less light to the more intricate considerations of its surface and solidity. In all which are discovered variety of delightful properties, that surprise our mind with unexpected truth and conviction, and lay a foundation to many noble and useful Rules of Practice, not onely in the several parts of Mathematics, but in many instances of common life.

Demonstration.

Demonstration therefore serving (as was said) to enlighten the mind, the Reader must take care in the perusal of these Theorems, not to put himself off with an obscure conception of the matter before him, and a fancy onely that he apprehends it. He must not leave it, till all be as clear as if writ with a beam of the Sun. And tho some of the Theorems may not be of such universal use as others, yet all will be profitable to

perfect the Mind and Reaſon, when they are thus clearly apprehended.

Directions for reading the following book. In order to which, the Reader muſt firſt endeavour to poſſeſs himſelf of the matter which is to be proved from the *Title* of each Propoſition, and then lay down plainly in his mind or paper, how much is ſuppoſed, and how much remains to be proved. He muſt conſider alſo that ſome things admit of a *direct* proof; as when a thing can be demonſtrated that it *is* ſo: others onely of an *indirect* proof, when it is demonſtrated that it *muſt be* ſo; becauſe of ſome abſurdity that would follow from the contrary if it were not ſo. He

Of the Axioms. muſt conſider that the *Axioms* are truths, which are ſo naturally clear, that they admit of no further proof but their own light; at leaſt if we duly underſtand the terms and words in which they are expreſt. And to give us an account of the

Definitions. meaning of thoſe terms and words, are the *Definitions* at firſt laid down, which we muſt ſettle well in our memories before we enter upon the book; at leaſt ſo much of them as relate to that particular part we undertake. To conclude, if any Propoſition ſeems difficult at the firſt peruſal, the Reader is not to be diſcouraged, or to pore too long upon it, but to paſs on to others which will probably furniſh him with light to apprehend what he omitted againſt a ſecond review.

Before we proceed to give inſtances of theſe Directions in ſome of the following Propoſitions, it may be neceſſary to give a few hints to ſuch as are wholly ſtrangers to the Mathematics. 1. That tho in Def. 2. we ſuppoſe a Line to be onely an imaginary thing, *a length without breadth*, yet it does not hinder but that we may expreſs it

by real lines on paper, as we do our thoughts by letters and words. 2. In Def. 6. the length of the sides [*a b, a c*] Fig. 1. signifies nothing to the greatness of the angle [*a*]; onely if you extend these sides further from one another, as when you open a two foot Rule, then the angle [*a*] is thereby increased. 3. When 3 letters are set to express an angle, the middlemost points it out. So if we say the $\angle$ *bac*, we mean the $\angle$ *a*. 4. Any side of a $\triangle$ may be call'd its *Base*, and then the angle opposite to this side is its *Top*. 5. By *(Constr.)* Construction, we mean the drawing of lines or framing of figures. For the further explication of Def. 37. add this; That is, which are of the same kind: for a line and a surface for instance can't be compared together, nor be said one to exceed the other. And of Def. 46. 'tis there meant, that not both the Antecedents and both the Consequents should be in one figure.

Of the I. Theor.

The I. Theorem, *that one line falling upon another, makes angles equal to* 2 ∟, is immediately founded upon an Axiom, and admits of a *direct* proof. And it is easie for the ingenious Reader to observe from hence, That all the angles that can be made about a point (as *b* Fig. I.) are equal to 4 ∟. For suppose a line to be run through this point (as *c d*) all the angles above the line will be equal to 2 ∟, according to the I. Theor. And by the same reason all below = to 2 more. And that the Reader is capable to make such a natural inference from any Theorem, is many times supposed in the following Discourse.

Theor. II. explained.

The II. Theorem is brought in upon this occasion. We often suppose (as a thing reasonable to be granted) that a right line already drawn may be continued onward, at either end, by joyn-

ing another piece to it. But when a piece is so joyned, it is sometimes requisite to be proved, that it is rightly joyned, so as (together) to make but one right line. Now this *Proposition* tells us a particular case, in which two pieces so joyned shall certainly be a right line. That suppose *cb* were drawn, and *db* added to it, then *cd* makes but one right line, if a line, as *ab*, falling upon their point of joyning, *b* makes angles with the line *cd* equal to 2 ∟. Here then is something supposed, and something to be proved. It is supposed that the ∠s above the point *b* are = 2 ∟, viz. *abc*, *abd*. And it is to be proved that the line *cd* (is well joyned, in *b*, so as to be but) one right line. Now this does not admit of a *direct* proof, onely from an absurdity that would follow if *cd* were not a right line. For if *bd* were not rightly joyned, it must have lean'd either more upward or more downward (as *be*) there is no third way to be thought on. Put case then, that either of these *cbe* be a right line (and *cd* not one;) this absurdity would follow, that the ∠ *abd* would be = to the ∠ *abe* i.e. a part would be = to the whole. For it was supposed (as we have already laid down) that the ∠s *abc*, *abd* were = to 2 ∟. And if *cbe* be a right line, it follows from the I. Theor. that the ∠s *abc*, *abe* are = to 2 ∟ also. Wherefore by *Ax*. 6. each pair of ∠s together *abc*, *abd* are = to the other *abc*, *abe* together; since either of these pairs are = to a third thing; namely to 2 ∟. And leaving out *abc*, which is common to both pairs (according to *Ax*. 8) there remains *abd* = *abe*, which is the absurdity we spake of. So that we can't but be satisfied that *cbd* is well joyned, and makes but one right line. Q. E. D. (*The thing to be prov'd.*)

This

This may be demonstrated as the foregoing *Th. XIII.* from *Ax.*4. by reversing the $\triangle$, and laying it (as it were) upon it self. But to avoid the too often repetition of this method, we chose rather to found it upon what we shall here add as a third Sect. to Def.6. which we shall likewise refer to in Theor.XIX.

Def.6.Sect.3. The measure of this inclination *Th. XIX.* (either an arch of a circle *BC*, of which the angle [*a*] is the centre, which we leave to Trigonometry; or else) a streight line [*gh*]; which being lengthened or shortened (the points *g*,*h* remaining still at the same distances from the angle *a*) will make the inclination of the lines *ab*, *ac* more or less than it is; but if the line *gh* remains the same and in the same place, the inclination will be the same. And by consequence if *IK* be of the same length with *GH*, and the points *I*,*K* at the same distances from the angle *D*, as *G*,*H* are respectively from *A*; then *DI*, *DK* are said to have the same inclination with *AG*, *AH*.

Figures being between the same Parallels are *Th. XXVI.* of the same height (Def.47.) because all Perpendiculars between the same Parallels are $=$. For this is included in the notion of parallel lines; which are supposed (in the Definition of them) to keep all the way an equal distance from each other.

The Doctrine of *Proportion*. Chap.III. is some- *Proportion.* what abstracted and nice, and requires a comprehensive mind to take in several things into one thought; wherefore there is the more need of an awakened attentiveness to the perusal of it, and particularly to be well acquainted with the Definitions and Characters belonging to it:

but

but when these once grow familiar, the great difficulty is over; and we are let loose into boundless field of demonstration and variety for *Proportion* confines not it self to Lines and Figures, but is applicable to Time, and Weight and Motion, and Force, and in short to whatsoever admits of *greater* and *less*.

Theor. LXXXIII. What is there said in the last line, that twice $\frac{4}{3}$ is the same as $\frac{16}{9}$, may be thus explained: For $\frac{16}{12}$ is $=\frac{4}{3}$, and $\frac{12}{9}$ is $=\frac{4}{3}$, (12 being 3 parts of 16, just as 3 is of 4, and as 9 is of 12.) So that $\frac{16}{9}$ is $=$ to twice $\frac{4}{3}$.

Th. XCIII. To make the fourth Sect. of this plain, let us call *Twice as many angles as there are sides*, by the name of *B*. Now I say that all the inward angles both at the sides and centre together, are $=$ to *B* (by 2.) And all the $\angle$s at the sides, both inward and outward together, are $=$ to *B* (by 3.) Therefore (by *Ax.* 6) all the inward $\angle$s, at the sides and centre together, are $=$ to all the $\angle$s at the sides, both inward and outward, together. Now if from these 2 equal sums you take away the common, viz. all the inward $\angle$s at the sides, there remains (by *Ax.* 8) all the inward $\angle$s at the centre, $=$ to all the outward $\angle$s at the sides. Which $\angle$s at the centre were $=$ to 4 $\llcorner$ (by 2.) Therefore the outward $\angle$s at the sides, are $=$ to 4 $\llcorner$ also (by *Ax.* 6.) Q.E.D.

From the former part of this Theorem we may frame a Rule, to know the quantity of the angle of any regular Polygon. For since all the $\angle$s together, are $=$ to twice as many $\llcorner$s as there are sides (except 4.) If from the double of the number of the sides we take out 4, and divide the remainder by the number of $\angle$s in the Polygon, the Quotient will give us the measure

of

of each angle. As for inſtance; In a regular Polygon of 12 ſides, the double of the ſides is 24; out of which 4 being taken, there remains 20, which (20) divided by 12 gives $1\frac{2}{3}$. So that each angle of ſuch a Polyg. is = to 1 and $\frac{2}{3}$ of a ∟.

The Doctrine of the *Power of Lines* is the foundation of a great part of *Arithmetic* and *Algebra*; particularly Theor. CV. gives the Rule for extracting the *Square Root*. *Power of Lines.*

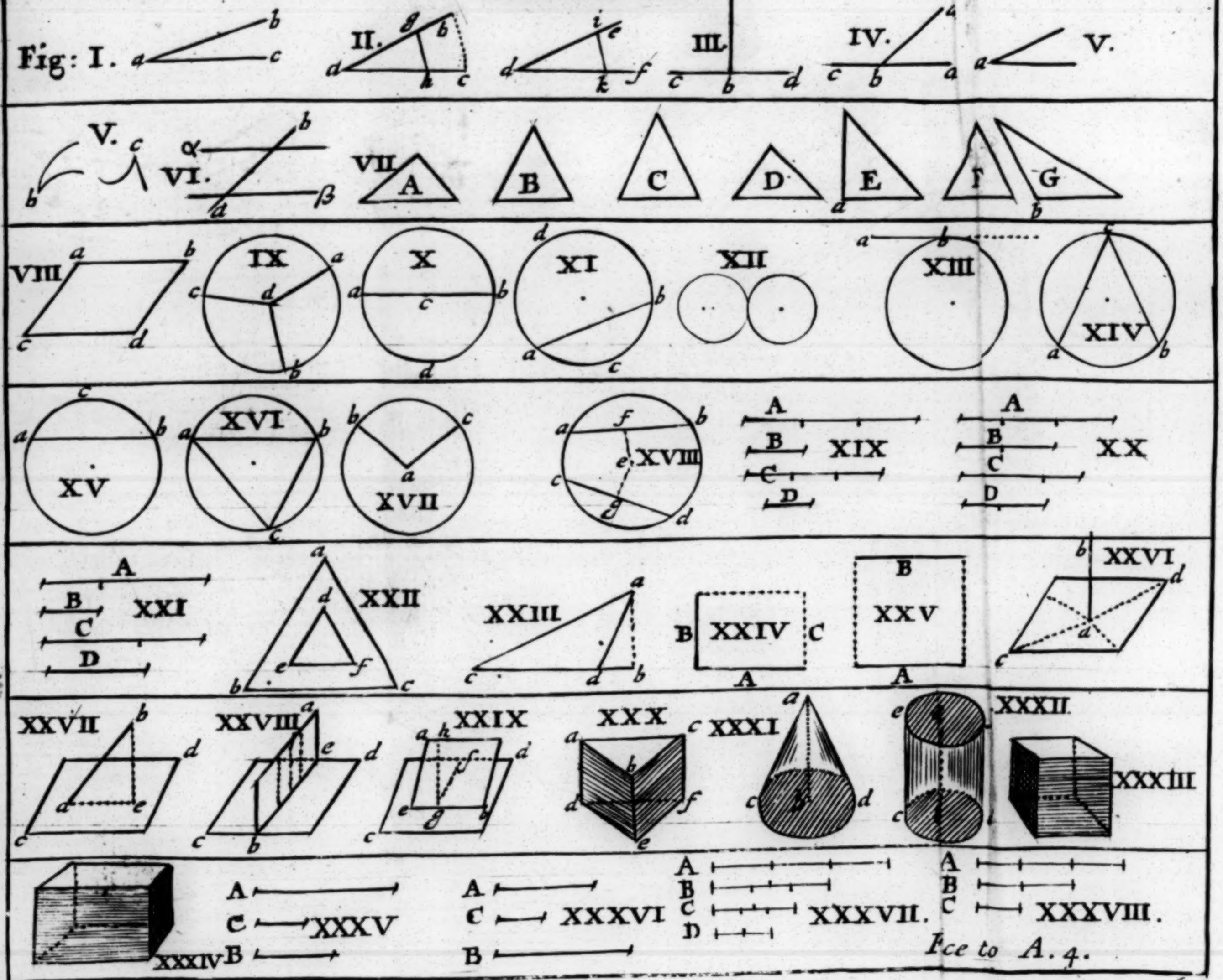
This note — between 2 letters, as *A*—*E*, ſignifies *E* ſubducted from *A*, and is to be read *A* leſs *E*. (= in the firſt line ſhould ſtand after *Æ* in the ſecond, ſo as to be read *Æ* is = to *ZE*—*Eq*. And the reaſon is becauſe *ZE* is = *Æ*+*Eq* (by CIV.) therefore *Æ* is = *ZE*—*Eq*. For if 8 ſuppoſe be = 6+2, then 6 is = 8—2. It is the ſame thing whether you take 2 from 8, or add 2 to 6, to make them equal. *Th. CVI.*

The concluſion of this Theorem, viz. that $2Q\frac{1}{2}A+E$ are $=2Q\frac{1}{2}A+4\frac{1}{2}Æ+2Eq$, may appear thus (from CV.) For one $Q\frac{1}{2}A+E$ is $=Q\frac{1}{2}A+2\frac{1}{2}A\times E+Eq$. (For in this caſe $\frac{1}{2}A$ is not look'd upon as a Fraction, but an entire quantity) therefore $2Q\frac{1}{2}A+E$ is = to all the reſt taken twice, viz. to $2Q\frac{1}{2}A+4\frac{1}{2}Æ+2Eq$. *Th. CX.*

To conclude, theſe *Elements* (ſerving according to the original of their name *eLeMeNta*, as a neceſſary Key or Alphabet to further progreſs in Demonſtration) are not to be ſlightly paſſed over, but imprinted in our minds and memories. And if we go on then to *Trigonometry*, which takes an eaſie riſe from hence, we ſhall find our ſelves confirmed more in the uſe, as well as ſatisfied in the uſefulneſs of them.

THEO-

Definitions



Face to A. 4.

THE

Elements *or* Principles OF GEOMETRIE.

DEFINITIONS.

CHAP. I.

Of Lines.

G*EOMETRIE is the Art of Measuring Bodies: In order to which end, it considers three principal things that belong to a Bodie, namely, Superfice or Surface, Lines, and Points: Which in their proper notion, do signifie, the* Out-side, *the* Edg, *and the* Corners *of a Bodie; and so, are real parts of it: But*

in the ſenſe that Geometrie uſes them, they do not properly belong to a Bodie, only are applied to it by our imagination. For if a Bodie be dipt in water, though the water touches it every where, yet we may fancy a diviſion between it and the water, which imaginary diviſion, is called a Surface. *And thus, every imaginary diviſion of a Bodie, begets a* Surface; *(between the parts ſo divided:) of a* Surface, *begets a* Line; *of a* Line *begets a* Point. *And ſince whatſoever divides a thing, terminates or bounds the parts ſo divided; therefore,*

1. *A* **Point** *is ſaid to be bound of a* Line.

2. *A* **Line** *the bound of a* Surface. *And,*

3. *A* **Surface** *the bound of a Bodie.*

The firſt, *having neither length nor breadth: The* ſecond, *length without breadth; and the* third, *length and breadth without depth.*

4. *A* **Right-Line**, *is that which lies ev'n between its two bounding Points; and by conſequence, marks out the ſhorteſt way from one Point to another.*

5. *A*

5. *A* **Plane Surface** *is that which lies ev'n, between its two bounding Lines, and because it is stretcht out strait (like the head of a Drum) therefore it is the shortest that can lie between the same two Lines.*

6. *An* **Angle**, *is the corner* [a] *that is made by the meeting of two Lines:* 1. *Which Angle is greater or less, according as the said Lines* [a b, a c] *lean nearer or stand further off from one another, which is call'd their* Inclination: 2. *So that if several Lines* [a b, a c, *and* d e, d f] *have the same* Inclination, *the Angle which they make* [a = d] *are equal.* Fig. I. Fig. II.

7. *A* **Right-Angle**, *is when one Line* [a b] *so meets another* [c d], *that the two Angles on each side,* [a b c, *and* a b d], *are equal to each other.* Fig. III.

Perpendicular: *In this case, the Line* A B, *is said to be Perpendicular to* C D.

8. *An* **Obtuse** *Angle* [a b c] *is that which is bigger than a* Right. Fig. IV.

9. *An* **Acute** *Angle* [a b d] *is that which is less than a* Right.

10. *A* **Right-Lin'd-Angle** [a] *between two Right-Lines, a* Curv-lin'd, [b] *a* mixt [c]. Fig. V.

11. **Parallel**

Fig. VI. 11. **Parallel** *Lines* [α β] *are those which lean not at all towards one another: So that if they were drawn out infinitely, they wou'd never meet.*

CHAP. II.

Of Figures. PART I.

12. A **Plane Figure**, *is a Plane Surface inclos'd in one or more Lines.*

Fig. VII. 13. *A* **Triangle**, *is a Figure bounded with three Lines*, A.

14. **Equilateral**, *which has all sides equal*, B.

15. **Isoscele**, *or equal legg'd, which has two sides equal*, C.

16. **Scalene**, *which has no sides equal*, D.

17. **Right-Angled**, *which has one Right-angle*, [a] E.

18. **Acute-Angled**, *which has all its Angles Acute*, F.

19. **Obtuse-Angled**, *which has one Obtuse Angle* [b] G.

20. *A*

 Parallelogram, *is a four sided* ... *whose two opposite sides* [a b, c d *or* ... *are Parallel Lines.* Fig. VIII.

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a Circle. PART II.

 Circle *is a plane Figure boun-ded with one Line, call'd the* ...rence [a b c a], *to which all the* ...a, d c, d b] *that can be drawn* ... *Point* [d] *in the middle of the* ... *are equal to one another; these* ... *call'd* Radius's, *and the Point* ...*dle, the* Center. Fig. IX.

he Diameter *of a Circle, is a* ...*e* [a b] *passing through the Center,* ...*ded at each end with the Circum-* ...*and dividing the Circle into two* ...*s.* Fig. X.

 Semicircle *is the Figure* [a d b] ... *between the Diameter, and half* ...*nference.*

 Segment *(of a Circle) is a* ...*ntain'd between a Right-Line* ... Chord) *and any part of a Cir-* ...*e,* [a c b, *or* a d b] *call'd an* Fig. XI.

25. Equal

Fig. VI. 11. **Parallel** *Lines* [$\alpha \beta$] *ar which lean not at all towards one a So that if they were drawn out in they wou'd never meet.*

CHAP. II.

Of Figures. PART

12. A **Plane Figure**, *is Surface inclos'd in more Lines.*

Fig. VII. 13. *A* **Triangle**, *is a Figure with three Lines*, A.

14. **Equilateral**, *which has equal*, B.

15. **Isoscele**, *or equal legg'd*, *two sides equal*, C.

16. **Scalene**, *which has no qual*, D.

17. **Right-Angled**, *which Right-angle*, [a] E.

18. **Acute-Angled**, *which h. Angles Acute*, F.

19. **Obtuse-Angled**, *which Obtuse Angle* [b] G.

20. *A* **Parallelogram**, *is a four sided Figure, whose two opposite sides* [a b, c d *or* a c, b d] *are Parallel Lines.* Fig. VIII.

Of a Circle. PART II.

21. A **Circle** *is a plane Figure bounded with one Line, call'd the* Circumference [a b c a], *to which all the Lines* [d a, d c, d b] *that can be drawn from one Point* [d] *in the middle of the Figure, are equal to one another; these Lines are call'd* Radius's, *and the Point in the middle, the* Center. Fig. IX.

22. *The* **Diameter** *of a Circle, is a Right-Line* [a b] *passing through the Center,* [c] *bounded at each end with the Circumference, and dividing the Circle into two equal parts.* Fig. X.

23. *A* **Semicircle** *is the Figure* [a d b] *contain'd between the Diameter, and half the Circumference.*

24. *A* **Segment** (*of a Circle*) *is a Figure contain'd between a Right-Line (call'd a* Chord) *and any part of a Circumference,* [a c b, *or* a d b] *call'd an Arch.* Fig. XI.

25. **Equal**

25. **Equal Circles**, *are such, whose Diameters or Radius's (that is Semidiameters) are eqnal.*

Fig. XII. 26. **Circles** *are said to* Touch, *when they do only touch, and not cut one another.*

Fig. XIII. 27. *A* **Right-Line** [a b] *is said to touch a Circle, when being continu'd it does not cut the Circle: This is call'd a* Tangent.

Fig. XIV. 28. *An* **Angle** [c] *is said to* stand upon *that part of a Circumference* [a b] *which is opposit to it.*

Fig. XV. 29. *An* **Angle of the Segment** [a b c] *is made by the Circumference and a Right-Line cutting it.*

30. *An* **Angle in the Segment** [c] *is made by two Right-Lines* [a c, b e] *rising from the* Angles of the Segment, *and meeting in the Circumference.*

Fig. XVI. 31. *An* **Angle** *of* Contact, [b] *is between the Tangent and Circumference.* F. 13.

Fig. XVII. 32. *The* **Sector** *of a Circle,* [a b c] *is a Figure made by two Radius's and part of the Circumference.*

Fi. XVIII. 33. **Right-Lines** [a b, c d] *are said to be* Equidistant *from the Center,* [e], *when Lines* [e f, e g] *drawn Perpendicular from the Center to them, are equal.*

CHAP. III.

CHAP. III.

Of Proportion.

34. A **Multiplied** *Magnitude*, [a] *is that which contains another Magnitude*, [b] *a certain number of times precisely*. Fig. XIX.

35. *An* **Aliquot** *part*, *or simple*; [b] *which being repeated a certain number of times, equals, or measures out another* [a] *Magnitude precisely.*

(1.) Like Aliquot *parts*,[b, d] *are such, as being equally repeated do measure out their respective wholes*, [a, c.]

(2.) Like Parts, *are those that are equally contain'd in their respective wholes: Thus* B *and* D *are* like parts, *because* B *is contain'd once and a half in* A, *and* D *once and a half in* C. Fig. XIX. Fig. XX.

36. **Ratio** *or* Reason, *is the comparison of two quantities* [a, b] *one with another; whereby, one is said to be bigger or less than the other: In which comparison, that which precedes*, [a] *is call'd the* Antecedent, *and the other* [b] *the Consequent.* Fig. XIX.

37. *Those*

37. *Those* **Quantities** *only admit of a* Reason, *which being Multiplied may exceed each other.*

Fig. XIX. 38. *The* **Reasons** *(between* A, B, *and* C D) *are said to be the* same (equal, *or* like) *when both the consequents* (B *and* D) *are* like parts *of their respective Antecedents* (A *and* B). *That is, since no quantity can be said to be big or little, but as it is compar'd to another, therefore if* B *and* D *are like parts of* A *and* C, *then* A *is said to be as big, in respect of* B, *(or, to have the* same reason *to* B) *as* C *has in respect of* D: *Thus exprest,* A . B :: C . D, *or thus,* $\frac{A}{B} = \frac{C}{D}$.

Fig. XXI. 39. *One* **Reason** *is said to be greater or less than another, when one of the Consequents* [b] *is more exceeded by its Antecedent* [a], *than the other* [d] *by its Antecedent* [c]. *That is,* A *is bigger in respect of* B, *than* C *is in respect of* D, *or, the* Reason *of* A *to* B, *is bigger than the reason of* C *to* D, *which is thus exprest;* $\frac{A}{B} \sqsubset \frac{C}{D}$.

40. *The*

40. *The* Equality *of Reasons*, *(mentioned in* def. 38.) *is call'd* Proportion: *That is*, A, B; C, D *are said to be* Proportionals. *And in* def. 39. A, B; C, D *are* Unproportionals.

41. Continual *Proportionals*, *are when the middle term (or quantity) is taken twice: as when*, A *is as much bigger than* B; *as* B, *is than* C; *which is thus exprest*, A, B, C, $\div\!\!\cdot$

42. *In the foregoing case the reason of the first term to the third*, *is said to be* Duplicate *to the reason of the first to the second*; *or of the second to the third*, *(for both these* reasons *are the same*, *by* def. 38.) *and if there be more terms added*, viz. A, B, C, D, *&c.* $\div\!\!\cdot$ *the Reason of the first* [a] *to the fourth* [d], *is said to be* Triplicate, *&c.*

43. *But if the terms* ABC *are not Proportionals (as in* def. 39.) *then the Reason of* A *to* C, *is said to be* compounded *of the Reason of* A *to* B, *and of* B *to* C, *thus exprest*, $\frac{A}{C} = \frac{A}{B} + \frac{B}{C}$.

44. *The* Homologous *terms in any Proportion*, *are the two* Antecedents, *or the two* Consequents: *Thus* A, C, *and* B,

D,

D, (*in* Fig. 20.) *are the* Homologous *terms.*

Fig. XXII. 45. *Like* **Right-Lin'd** *Figures,* [a b c, d e f], *are such as have equal Angles, and the sides about those equal Angles, Proportional, as if the Angle* B, *be supposed to be equal to the Angle* E, *then the sides* AB . BC :: DE . EF, *and so of the other Angles and sides.*

46. **Reciprocal** *Figures, are when you compare the sides of one Figure to the sides of the other, and the Antecedents and Consequents of the Reasons, are in both Figures.*

Fig. XXIII 47. *The* **Height** *of any Figure, is a Perpendicular Line* [a b] *drawn from the top of it* [a] *to the basis* [c d].

CHAP. IV.

Of the Power of Lines.

Fig. XXIV 48. A **Rectangle** *is a Parallelogram* (def. 20.) *whose Angles are Right.*

Fig. XXV. 49. *A* **Square** *is a Rectangle that has all its sides equal, these are also call'd the Powers of Lines.*

CHAP. V

CHAP. V.

Of Surfaces.

50. A **Right-Line,** [a b] *is* Right *(or Perpendicular) to a* Plane [c d] (def. 5.) *when all the Lines* [a c, a d, *&c.*] *that can be drawn from that Point* [a] *where it touches the Plain, upon the ſaid Plane do make Right-angles with it,* viz. *when* CAB, DAB, *&c. are Right-angles* Fi. XXVI.

51. *The* **Inclination** *of a Right-Line* [a b] *to a Plane,* [c d] *is meaſured by the Angle* B A C, *where the Line that leans, and the Perpendicular, do touch the Plane.* F. XXVII.

52. *One* **Plane** [a b] *is* Right *to another Plane,* [c d] *when all the Lines in the firſt Plane* [a b] *that are Perpendicular to the common ſection,* [b e] *are alſo Perpendicular to the other Plane* [c d] (def. 50.) F. XXVIII.

53. *The* **Inclination** *of one Plane* [a b] *to another* [c d] *is meaſured by the Angle* [f g] *between two Lines* [h g, f g] *in each Plane, which are Perpendicular to the common interſection* [e b]. Fig. XXIX.

54. *This* **Inclination** *will be* equal, (*or* like) *in several Planes when this Angle is equal.*

55. **Parallel** *Planes are such as have no Inclination to one another.*

56. *A* **Solid** *Angle, is a Corner made by the meeting together of several Plan Angles (three at the least) in one Point.*

CHAP. VI.

Of Solids, or Bodies.

57. A **Solid** *or* Body, *is that whic has Length, Breadth, an Depth.*

58. Like **Solid** Figures, *are such are contained under an equal number like plain Figures.*

59. Equal and like **Solid** Figures, *a such as are contain'd under an equal numb of like and equal plain Figures.*

60. *A* **Pyramid**, *is a Solid Figure whose sides are plain Triangles, their seven tops meeting together in one Point*

61. *A* **Prism**, *is a Solid Figure, the two opposite sides (or ends)* [abc, def] *of which are* like, *equal, and Parallel; and all the other sides* [abde, *&c.*] *are Parallelograms.* Fig. XXX.

62. *A* **Sphere** *is a Solid Figure bounded with one Surface; to which (Surface) all the strait Lines that can be drawn from one Point within the Figure, call'd the* Center, *will be equal.*

63. *The* **Axis** *of a* Sphere, *is that resting Right-Line, about which, if a Semicircle be turn'd, it will beget a Sphere.*

64. *The* **Center** *of a Sphere, is the middle of this* Axis.

65. *The* **Diameter** *of a Sphere, is a Right-Line passing through the Center, and bounded at each end in the Surface of the Sphere.*

66. *A* **Cone**, *is a Solid Figure, rising from a Circular base* [cd] *according to Right-Lines,* [ca, da] *and ending in a Point,* [a] *which is call'd the Vertex, or Top.* Fig. XXXI.

67. *The* **Axis** *of a Cone is a Right-Line drawn from the Top,* [a] *to the Center* [b] *of the base* [cd].

68. *A* **Right Cone.** *See* def. 71.

F.XXXII. 69. *A* **Cylinder** *is a Solid Figure, rising from a Circular base* [c d] (*or Circle*) *according to Right-Lines,* [c e, d f] *and ending in an equal Circle* [e f].

70. *The* **Axis** *of a Cylinder, is a Right-Line,* [a b] *joyning together the Centers of the two Circles.*

71. *A* Right **Cone** *or* **Cylinder**, *is when the Axis is Perpendicular to the base.*

72. Like **Cones** *or* **Cylinders**, *are such, whose Axes, and the Diameters of their bases, are proportional.*

F.XXXIII. 73. *A* **Cube** (*or Dye*) *is a Solid Figure contain'd under six equal Squares.*

74. *A* **Tetraedrum**, *contain'd uuder Four Triangles, equal and equal sided.*

75. *An* **Octaedrum**, *contain'd under Eight Triangles, equal and equal sided.*

76. *A* **Dodecaedrum**, *containd under Twelve Pentagons, equal and equal sided.*

77. *An* **Icosaedrum**, *contain'd under Twenty Triangles, equal and equal sided.*

These five last only, are call'd *Regular* Bodies.

F.XXXIV. 78. **Parallelepipid** (P pp) *is a Solid Figure, contain'd under six Parallelograms, the opposite of which, are Parallel.*

79. *One*

79. *One Figure is ſaid to be* Inſcrib'd *in another, when all the Angles of the Figure inſcrib'd, touch either the Angles, Sides or Planes, of the other Figure.*

80. *A* Figure *is* Conſcrib'd (*or* Circumſcrib'd) *when either the Angles, Sides or Planes of the outward Figure, touch all the Angles of the Figure that is inſcrib'd.*

Note, That theſe *Definitions*, though put all together, for the convenience of references, will be beſt perus'd ſeverally, before each reſpective *Chap.* in the Book, to which they belong.

AXIOMS.

1. *Between two Points, there cannot lye more than one Right-Line, (nor between two Lines, more than one Right Surface) but they will be* Coincident; *that is, become one. Therefore ſuch Lines (and Surfaces) can't have one common Segment, that is, one part or portion, common to two or more of them.*

2. *All* Right Angles *are equal to one another.*

3. Parallel Lines, [α β] (having no Inclination to one another) have the same Inclination to a third Line [β].

4. Those things which being laid upon one another, do meet in all parts, are equal. The Converse of this (to wit, that equals being laid upon one another, will meet) is true in Lines and Angles, but not in Figures.

5. A Whole is greater than any part of it; and equal to all its parts taken together.

6. Things that are equal to a third, are equal to one another: If A be equal to C, and B, be eqnal to C, then A is equal to B.

7. The halfs (or doubles) of things that are equal, are equal, and so are any multiples, &c.

8. If you add, or take away equal parts from things that are equal, the remainders will be equal.

9. If you add, or take away equal parts from things that are unequal, the remainders will be unequal.

Of Proportion.

10. *Those things that have the same* Reason *to a third thing, (or to things that are equal,) are themselves equal: That is, those things that are equally great in respect of another, are equal between themselves; and convertedly.*

11. *And if* A, *be greater in respect of* C, *than* B *is in respect of* C, *then* A *is greater than* B. Fi.XXXV.

12. *Or (which is the same in other words)* A *is less than* B, *if the* Reason *of* C *to* B, *be greater* (def. 39.) *than the* Reason *of* A *to* B: *That is, if* A *be less in respect of* C, *than* B *is, in respect of the same* C, *then* A *is less than* B. F.XXXVI.

13. *Those* Reasons *that are* equal *(or* like) (def. 38.) *to a third* Reason *are equal between themselves, and those that are unequal to a third, are unequal to all that are equal to this third.*

14. *A* Whole, *and all its parts together, have the same* Reason *to a third thing.*

$=$	Equal to.
$\sqsubset$	Greater.
$\sqsupset$	Leſs.
$+$	Added to.
$-$	Subtracted, or divided by.
$\times$	Multiplied by.
$::$	Like.
$\div\!\div$	Continued Proportion.
$\llcorner$	Right-Angle.
$\angle$	Angle, or Acute-Angle.
$\triangle$	Triangle.
$\bigcirc$	Circle.
$\square$	Square.
$\sqsubset\!\sqsupset$	Rect-angle.
Perp.	Perpendicular.
Pll.	Parallel.
Pgr.	Parallelogram.
Ppp.	Parallelepipedon.
Hyp.	Hypotheſis, or Suppoſition.
Pre:	Precedent Propoſition.

THEO-

THEOREMS.

[Note. *In these* Theorems *(or speculative Principles) it is enough to suppose that Parallel Lines can be drawn, Angles fram'd, and Circles describ'd. The manner how to do them, will be shew'd and Demonstrated in the* Problems *(or Practical Principles) that come after.*]

CHAP. I.

Of Lines. PART I.

Theorem I.

One right Line [ab] *falling upon another* [cd] *makes either two* ∟s, *or such as are* = 2 ∟.

IF AB stands Perp. it makes [a] 2 ∟s ABC, ABD. If it lean, (as EB,) the Angles EBC. EBD, take up the same place as the former 2 ∟ ones had done, and by consequence [b] are equal to them. *Q. E. D.*

[a] *Def.* 7.

[b] *Ax.* 4.

II.

Theorem II.

That [c d] *is a right Line, on which* … *other* [a b] *standing, makes* 2 ∟s *such as are* = 2 ∟.

IF CD, be not a right Line, then the r… Line will fall either under or over it, …
c Sup. CBE. Now the Angles ABC, ABD = [c] (…
d Præ. = [d]) ABC, ABE, *i. e.* a part equal to … whole, *Q. E. A.* Therefore C D will be … right Line. *Q. E. D.*

Theorem III.

The opposite Angles of crossing Lines (*c*… Head *Angles*) *are equal to one anot*… *as* a = b, *and* c = d.

e I. D + A = [e] (2∟ = [e]) D + B, theref… (D being common) A is = B. …
f Ax. 9. A + C = [f] (2∟ = [f]) A + D, therefor… being common) C is = to D. *Q. E. D.*

The reason is this, If HF cut EG Perp… the Angles are ∟ (by I) and therefore eq… (Ax. 2.) so then, C = D and A = B. N… if the point O remaining fixt, H F be le… aside, F will come to E at the same time … H comes to G, therefore whatsoever is lost … of the Angles B and A, is equally gain'd by …

oth…

...er Angles C and D, so [g] that B will always equal to A and C = D. Q.E.O.

[g] *Ax.* 8.

Theorem IV.

...right Line [γ δ] *cutting Parall.* [α β] *makes all the opposite Angles equal,* viz.
a = b = c = d. e = f = g = h

FOr *αβ* have [g] the same inclination to γδ, and therefore [h] make = Angles with it. ...at is, A = C and F = H. But A [i] = B and ... [k] = D, also E = F and G = H, and there...e [k] they are all = one another, viz. A = B ... C = D, and E = F = G, = H. *Q.E.D.*

[g] *Def.* 11.
[h] *Def.* 6.
[i] *Pre.*
[k] *Ax.* 6.

Theorem V.

...nd two of the opposite Angles (whether internal, [f c. bg] *or external)* [a h, c d] *are equal to* 2 ∟.

FOr, F. A (=[l] C) =[m] 2 ∟
B. E (=[l] G) =[m] 2 ∟ Therefore
C = } 2 ∟, and so of the rest. *Q.E.D.*
G = }

[l] *Pre.*
[m] *I.*

VI.

Theorem VI.

If the opposite Angles [abcdefgh] *equal, the Lines* $\alpha\ \beta$ *are* P ||.

n *Sup.* Since the Angle A = [n] C, the Lines
o *Def.* 6. have the same [o] inclination to a third
p *Ax.* 3. $\gamma\ \delta$; and therefore [p] no inclination to on
p *Def.* 11. nother, that is [q] to say, are Parallels. *Q. E.*

Theorem VII.

Also if the opposite Angles are = [fc, *or* bg, *or* ah, *or* ed,] *the Line are Parallels.*

r *Sup.* For, FC = [r] (2 ∟ = [s]) FA, there
s *I.* taking away F which is common, there
t *Ax.* 8. mains [t] C = A. Wherefore [u] $\alpha\ \beta$ are Paral
u *Pre.* *Q. E. D.*

Theorem VIII.

Lines Parall. $\alpha\ \beta$ *to a third,* δ *are Par to one another.*

If you deny it, let α be suppos'd inclin'd
w *Ax.* 3. β, therefore [w] δ is inclin'd to β after

sa

e manner, contrary to our suppoſition, ch is abſurd; therefore α is not inclin'd to nd by conſequence is Parall. to it. *Q. E. D.*

CHAP. II.

Of Figures. PART. I.

Theorem IX.

Triangle [a b c] *the external Angle* c b] *is equal to the two internal op-ſite Angles* [a b.]

$\alpha\,\beta$ be drawn Parallel to AB. Then the ngle D $=$ [x] A, and E $=$ [x] B. Therefore both together, are equal to A $+$ B. *Q.* [x] *IV.* [y] *Ax.* 5.

Theorem X.

three Angles of every Triangle, are equal to two right Angles.

, CD $=$ [z] 2 ∟ and D $=$ [a] A $+$ B, there- [z] *I.* re CAB $=$ 2 ∟. *Q. E. D.* [a] *Pre.*

XI.

Theorem XI.

Two sides [a b, b c] *of any Triang*
greater than the third [a c.]

[b] *Sup.* [c] *Def.* 4. For AC being [b] a right Line, marks
the shortest way between the points
C, and therefore is shorter than A B C,
lye between the same points. *Q. E D.*

Theorem XII.

In a Triangle [b c d] *equal Angle*
are subtended by equal sides [b c, b

Suppose the whole Triangle to be tur
backside forward, (represented by
and laid on the foreside B C D, so that t
ᗡ may fall upon C, and Ↄ upon D ; now
[d] *Sup.* the Angle C = [d] D, therefore [e] the A
[e] *Ax.* 4. shall meet exactly with C, and Ↄ with
by consequence the Line ᗡ ᙠ shall fall up
and Ↄ ᙠ upon D B. Lastly, the point
fall upon B, (for should it fall any
else, as in E, then the Lines ᗡ ᙠ and Ↄ
not fall upon C B and D B, against what
[f] *Ax.* 1. *and* 4. demonstrated) therefore C B = [f] D B.

Theorem XIII.

In a Triangle equal ſides [ab, bc] *ſubtend equal Angles.* [ac]

BEcauſe the Lines BA = [g] BC, therefore both of them in the point B, are equally diſtant from the Angles A and C, as are alſo both of them in their points A and C, (for the line AC is a common meaſure to both their points;) ſince therefore AB and BC, are equally diſtant from the Angles A and C in both their extream Points, and are equal [h] to one another, it follows [i] that the Angles ſubtended by them, A and C, are alſo equal. *Q.E.D.*

[g] *Sup.*
[h] *Sup.*
[i] *Def.* 6. *Caſ.* 3.

Theorem XIV.

In a Triangle the greater ſide [bc] *ſubtends the greater Angle.* [a]

IF not, let the Angle C be ſuppos'd = to A, then [k] AB will be = to CB againſt the ſuppoſition. *Q.E.A.* Suppoſe then C ⊏ A, and take the Angle ACD = A, therefore [k] the ſide CD = AD and adding DB to both, CDB will be = [l] to ADB. But CDB ⊏ [m] (CB ⊏ [n]) ADB. *Q.E.A. viz.* that the ſame CDB ſhould both = to and ⊏ than ADB.

[k] *XII.*
[l] *Ax.* 8.
[m] *XI.*
[n] *Sup.*

XV.

Theorem XV.

In a Triangle the ⊏ *Angle* [a] *is subtend*
by the ⊏ *side.* [b c]

IT is suppos'd that the Angle A is ⊏ than and it must be prov'd that the side B C is BA; for first, if B C were = B A, then Angle A would be = [o] C, contrary to the Su position. *Q. E. A.* Or 2*dly*, if B A were ⊏ B then the Angle C [p] would be ⊏ A, against Supposition also. Since therefore B C is neit = nor ⊐ B A, it follows that it must be B A. Q. E. D.

[o] *XIII.*

[p] *Pre.*

Theorem XVI.

In two Triangles if the sides be =,
Angles are =.

FIrst I prove the Angle A = D, for since sides A B, A C are = to D E, D F, and subtendent B C = E F, it follows [q] that the side B, A C, and D E, D F, have the same inclinat and by consequence that the Angles which t contain, A and D, are =. *Q. E, D.*

The same proof will serve for the other gles.

[q] *Def.* 6.

Theorem XVII.

In two Triangles, where two sides [b a c and e d f] and one Angle [a d] (between those sides) are =, all the rest, both sides and Angles, are =.

Since the sides BAC and EDF, are =; and also the Angles A and D, therefore be- g laid upon another, both the sides and Angles all exactly meet, [r] and by consequence the oint E shall fall upon B, and F upon C; there- re [s] the line EF shall exactly meet with BC, d by consequence[r] be = to it: and therefore[t] all e =. *Q. E. D.*

[r] *Ax.* 4.
[s] *Ax.* 11
[t] *Pre.*

Theorem XVIII.

n an equal legg'd Triangle (def. 15) a line [b d] drawn from the top, and cutting the base [a c] in the middle [d], is perp. to the same base.

Or in the two Triangles, ABD and BDC, the side BA = [u] BC, and AD = [u] DC, and D is common to both, therefore [w] the respective gles are = to one another; and particular- those two at D, which are therefore [x] ⌞; and consequence [y] BD is perp. to AC. *Q. E. D.*

[u] *Sup.*
[w] *XVI.*
[x] *I.*
[y] *Def* 9.

Theorem XIX.

In two Triangles, where two sides [b a c, e d f] *are* =, *and a third side* [e f,] ⊏ *t*[…] *Angle* [d] *subtended by this third sid*[…] *is also* ⊏.

THis follows manifestly from *def, 6. cas.* But may be demonstrated (if ne[…] were) as XVII, by laying one upon the […]ther.

Theorem XX.

The converse of the former. If two si[…] [b a c, e d f] *be* = *and one Angle* [[…] ⊏, *the subtendent* [e f] *of this Ang*[…] *shall be* ⊏ *also.*

IF you deny it, let the subtendent E F be =, […] ⊐ than B C, therefore the Angle D will […] = or ⊐ than A, contrary to the suppositio[…] wherefore it remains that the subtend. E F shou[…] be ⊏ than B C. *Q. E. D.*

Theor[…]

Theorem XXI.

iangles, if one ſide [a b, α β] *and gles* [α, a; β, b] (*adjoyning to this* ! =, *all the reſt ſhall be* =.

pon AB, and they ſhall meet [z]; alſo the [z] *Ax.* 4.
s α and β being [a] = to the Angles A, [a] *Sup.*
exactly meet [z] with them. Laſtly, The
ll fall upon C (for ſhould it fall any
as in E, then the ſide β δ would not
e ſide B C, as has been demonſtra-
efore [b] all the ſides are =, and by [b] *Ax.* 1.
: [c] the Angles. *Q. E. D.* [c] *XVI.*

Theorem XXII.

[a b c, α β γ] *that have two An-* a a, *and* β b,] =, *are equiangu- at is, have all their Angles* =.

e to prove that the Angle C = γ.
r the three Angles A B C together are
= [d]) α β γ together. Therefore if [d] *X.*
two equal ſumms, you take away A
B = [e] β, there will remain [f] C = γ. [e] *Sup.* [f] *Ax.* 8.

Theorem XIX.

In two Triangles, where two sides [b
are $=$, *and a third side* [e f,
Angle [d] *subtended by this t*
is also $\sqsubset$.

This follows manifestly from *de*
But may be demonstrated
were) as XVII, by laying one u
ther.

Theorem XX.

The converse of the former. I
[b a c, e d f] *be* $=$ *and one*
$\sqsubset$, *the subtendent* [e f] *of t*
shall be $\sqsubset$ *also.*

If you deny it, let the subtendent E
$\sqsupset$ than B C, therefore the Angl
$=$ or $\sqsupset$ than A, contrary to the f
wherefore it remains that the subten
be $\sqsubset$ than B C. *Q. E. D.*

Theorem XXI.

In two Triangles, if one side [a b, $\alpha\beta$] *and two Angles* [α, a; β, b] (*adjoyning to this side*) *be* $\doteq$, *all the rest shall be* $\doteq$.

SEt $\alpha\beta$ upon AB, and they shall meet [z]; also the Angles α and β being [a] $\doteq$ to the Angles A, and B, shall exactly meet [z] with them. Lastly, The point δ shall fall upon C (for should it fall any where else, as in E, then the side $\beta\delta$ would not fall upon the side B C, as has been demonstrated;) therefore [b] all the sides are $\doteq$, and by consequence [c] the Angles. *Q. E. D.*

[z] *Ax.* 4.
[a] *Sup.*
[b] *Ax.* 1.
[c] *XVI.*

Theorem XXII.

Triangles [a b c, $\alpha\,\beta\,\gamma$] *that have two Angles,* [[α a, *and* β b,] $\doteq$, *are equiangular; that is, have all their Angles* $\doteq$.

WE are to prove that the Angle C $\doteq \gamma$. For the three Angles A B C together are $\doteq$ [d] (2 $\llcorner$ $\doteq$ [d]) $\alpha\beta\gamma$ together. Therefore if from these two equal summs, you take away A $\doteq$ [e] α and B $\doteq$ [e] β, there will remain [f] C $\doteq \gamma$. *Q. E. D.*

[d] *X.*
[e] *Sup.*
[f] *Ax.* 8.

Theorem XXIII.

In two Triangles, where one side, [a b, $\alpha\beta$ *and two Angles* [a, α, *and* c, δ] (*thoug*
not adjoyning to this side) *are* =;
are equal.

[g] *X.* THree Angles of S = [g] (2 ∟ = [g]) thr
Angles of O together. If now fr
these equal summs you take away equals, *viz.*
[h] *Sup.* = [h] α and C = [h] δ, there will [i] remain B =
[i] *Ax.* 8. therefore [k] (the side A B being = [h] $\alpha\beta$)
[k] *XXI.* are =. *Q. E. D.*

Theorem XXIV.

In two Triangles [a b c, d e f] *where two sid*
are =, *and one Angle* [a = d] (*though n*
between those sides (*v. XVII,*) *all are* =
provided that the other Angles be of t
same kind; viz. ∟, *acute, or obtuse Ang*

[l] *Ax.* 4. BEcause the Angle D = A, therefore bei
laid upon it, they will meet,[l] and the poi
[m] *Sup.* E will fall on B (because the line D E = [m] A
also the point F, on C: if not, F must fall eith
below or above, in G or γ. First, not above, for
the Angle F be ∟ or obtuse, then A C B m
[n] *XIII.* [m] be the same, which is = [n] B γ C; (for B γ i

E F, $=$ [m] B C:) therefore A C B and B γ C will be both ∟ or obtuse, which is contrary to Th. X. 2*dly*, Nor can F fall below, for let F, that is G, be ∟ or obtuse, it will be $=$ [n] to B C G (because B G (*i.e.* E F) $=$ [m] B C) that is, 2 ∟ or obtuse Angles in one Triangle B C G, contrary to X. *Lastly*, Let the the Angle F be acute, A C B must [m] be the same, therefore B C G is obtuse $=$ [n] B G C, contr. X. Or (above) let A γ B, *i.e.* F, be acute; therefore B γ C is obtuse $=$ [n] B C γ, contr. X: therefore E F will fall on B C. And the three sides being $=$, [o] all will be $=$. *Q. E. D.* [o] *XVI.*

Theorem XXV.

Of several lines that can be drawn from a point given [c], *to a Line,* [a b] *the shortest is a perp.* [c d] *And of the rest, the nearer to this the shorter.*

BEcause in the Triangle C D E the Angle D is ∟ [p] (and ⊏ [q] then either of the others) therefore the subtendent C E is ⊏ C D. Further, because C E D [q] is acute, therefore [r] C E F obtuse, and thereupon [s] C F ⊏ C E. *Q. E. D.*

[p] *Sup.*
[q] X.
[r] *I.*
[s] *XV.*

Theorem XXVI.

Triangles [c a e, c b e] *upon the same b*[…] [c e] *and of the same height (def. 4*[…] *(or, which is the same, between the sa*[…] *Parallels* [a b, c d], *) are* =.

DRaw E F and B D Parallel to A C, [B F …] cause the Angle $\left\{ \begin{array}{l} CEA = {}^{t}EAF \\ CAE = {}^{t}FEA \end{array} \right\}$ [… th…]

t *IV.*

u *XXI.* the side A E is common, therefore u the T[…] angle C A E = E A F. In like manner the T[…] angle E F B, = E D B and C D B = C A[…] Therefore A C E + E D B ($\frac{1}{2}$ of the whole, […] D B) = E D B + C B E; and taking awa[…]

w *Ax.* 8. D B which is common to both, there rem[…] w A C E = C B E. *Q. E. D.*

Theorem XXVII.

Lines [a b, c d] *are Parallels, if e*[…] *Triangles* [c a d, c b d] *upon the s*[…] *base* [c d] *can stand between them.*

IF C D be not parallel to A B, then the Para[…] must fall either above or below A B, as in[…]

x *Præ.* Draw out C B to F, and joyn F D. There[…]

y *Sup.* the triangle C F D = x (C A D = y) C B[…] *Q. E. A.*

Theor[…]

Theorem XXVIII.

Triangles [S X] *upon* = *bases* [e f. e d], *between the same Parallels* [a b. c d], *are* =.

LEt the triangles S and X be so plac'd, that A B may be = C F or E D, then joyn A E and B F. Now because C F ≐ A B, and A F is common, and the Angle A F C = [z] F A B, therefore the Triangle S = A F B; in like manner [x] = A E B, and A E B = [b] A F B; so then S = (A F B = A E B =) X. *Q.E D.*

[z] *IV.*
[a] *XVII.*
[b] *XXVI.*

Theorem XXIX.

Those Lines [a b, c d] *are Parallel, which have between them equal Triangles* [c a e, e b d] *standing on equal bases.* [c e, e d]

FOr if A E be not Parallel to E D, the Parallel will fall either above or below; which cannot be, as in XXVII; therefore &c. *Q.E.D.*

Theorem XXX.

A Parallelogram [abcd] *is divided* [in] *the middle by the Diameter* [cb] [and] *the opposite sides are equal.*

[c] IV. BEcause the Angle $\left\{\begin{matrix} CBA = {}^{c}BCD \\ BCA = {}^{c}CBD \end{matrix}\right\}$
[d] XXI. the line C B is common, therefore [d] the Tri[an]gle C A B = C B D and by consequence [the] Pgr. is divided into two = parts. And bec[ause] of the = Triangles, the side A B = C D [and] A C = B D. *Q. E. D.*

Note, The opposite Angles [*a d*, and *c b*] [in] a Pgr. are =.

Since the two Triangles A B C and C B D [are]
[e] XVI. =, therefore [e] the respective Angles are [=] *viz.* A = D, and the 2 at C = 2 at B. *Q. E.* [D.]

Theorem XXXI.

Two Diagonals [a d, c b] (*or Dia[me]ters*) *in a Pgr. cut themselves in the m[id]dle* [e].

BEcause in the Triangles A E C and B E D [the]
[f] IV. Angle E B D = [f] E C A, and E D B = [E]
[g] XXX. A C and the side A C = [g] B D, therefore [h] [the]
[h] XXI. Triangles are =, and by consequence the [side] A E = E D, and B E = E C. *Q. E. D.*

Theorem XXXII.

A Line [a b] passing by the middle [e] of the Diameter [d c] cuts the Pgr. into two equal parts.

BEcause O + Z = [i] X + S, and Z = [k] X, [i] *XXX.*
(for the Angle E C A = [l] E D B, and E A C [k] *XXIII.*
= [l] E D B, and the side E C = [m] E D) there- [l] *IV.*
fore [n] taking away the equals Z and X, there [m] *Sup.*
remains O = S, which equals being added, it [n] *Ax.* 8.
will be O X = S Z. *Q. E D.*

Theorem XXXIII.

In a Pgr. the Complements (so S and Z are call'd) are =.

FOr O Z A = [o] X S V, all together. Also, [o] *XXX.*
O = [o] X and A = [o] V, therefore [p] there [p] *Ax.* 8.
remains Z = S. *Q. E. D.*

Theorem XXXIV.

Pgrs. [a d, c e, f g] between the same Parallels [a f, c h] upon the same [c d], or equal [c d, g h] bases, are equal.

WE are to prove that A D = C E = F G,
thus, The Triangle C B D ($\frac{1}{2}$ [q] A D and [q] *XXX.*
of

q *XXVIII.* of CE) = q EGH ($\frac{1}{2}$ of FG.) Therefo
r *Ax.* 7. r the wholes, AD, CE and FG are, also equ
Q. E. D.

CHAP. II.

Of a Circle. PART. II.

Theorem XXXV.

A Line [b d] ***passing the Center, is perp dicular to a Chord*** [a c] ***which it divi in the middle*** [d].

r *Radius's Def.* 21. s *Sup.* t *XVI,* u *I.*

FOr the side EA = r EC and DA = s DC } and DE is co mon, therefore t the Angles at D are =, and consequence u ∟. *Q. E. D.* (*Def.* 7.)

Theorem XXXVI.

And if it [b d] ***be perpendicular, it vides the Chord*** [a c] ***in the middle.***

w *Radius's.* x *Sup.* y *XXIV.*

FOr the side EA = w EC, ED is common, the Angles at D are = x, therefore AD y DC. *Q. E. D.*

Theor

Theorem XXXVII.

f it [b d] *divides a Chord* [a c] *in the middle, and be also Perpendicular to it, it passes through the Center.*

IF not, let E D pass though the Center, therefore the Angle E D C [z] is ∟ = [a] B D C that , a part = to the whole. *Q. E. A.*

[z] *XXXV.*
[a] *Sup.*

Theorem XXXVIII.

hat point [a] *is the Center of a Circle, from whence more than two equal* [a b, a d, a c] *right Lines can be drawn to the Circumference.*

DRaw the Lines BD, DC divided in the middle by AE, AF, because the sides BE = [b] ED, AB = [c] AD } and AE is common, therefore he Angles at E are =, and by consequence [d] ∟; herefore [e] EA passes the Center. As in like nanner, by the same reason does F A. Since herefore both these pass the Center, It must be here they meet; *viz.* in A. *Q. E. D.*

[b] *Construct.*
[c] *Sup.*
[d] *I.*
[e] *Pre.*

Theorem

Theorem XXXIX.

Crossing Chords [a c, b d] (*not passing* … *Center*) *do not cut each other in* … *middle.*

FOr if E were the middle of both, then … (passing the Center) would make F E C …
[f] XXXV. [f] to A C, and F E D $\llcorner$ [f] to B D; that is F…
[g] Ax. 2. = [g] F E D a part = to the whole. *Q. E. A*…

Theorem XL.

Of several Lines [a c, a d, a e] *dra*… *from one point* [a] (*in a Circle*) *to* … *circumference, the greatest* [a c] *pa*… *the Center*; *the rest are* $\sqsubset$ *as neare*… *this.*

[h] Radius's. 1. BC = [h] B D, therefore A B C (= [i] A B…
[i] Ax. 8. $\sqsubset$ [k] A D. *Q. E. D.*
[k] XI. 2. B F D $\sqsubset$ [k] B D (= [h] B F E, and omitt… the common B F) F D $\sqsubset$ [i] F E; and (adding… common A F) A D $\sqsubset$ [i] (A F E [k] $\sqsubset$) A… *Q. E. D.*

Theorem XLI.

If several Lines [a b, c d, a e] *drawn from one one point* [a] (*without a Circle*) *to the inner circumference* [b d e] *the greatest* [a b] *passes the Center; the rest are greater as nearer.*

AC + (CD =[l]) CB $\sqsubset$[m] AD. *Q. E. D.*

2. CFD $\sqsubset$[m] (CD =[l]) CFE; therefore (omitting the common CF) FD $\sqsubset$[n] FE, and (adding the common FA) DFA $\sqsubset$[n] (EFA $\sqsubset$[m]) EA. *Q. E. D.*

[l] *Radius's.*
[m] *XI.*
[n] *Ax.* 8.

Theorem XLII.

If several Lines [a b, a d, a e] *drawn from one point* [a] (*without the Circle*) *to the outward circumference,* [b d e] *the least* [a b] *is that which being continued would pass the Center, the rest are less as nearer to this.*

CDA $\sqsubset$[o] CBA, therefore (omitting the equals [p] CD, CB) DA $\sqsubset$[q] BA. *Q. E. D.*

[o] *XI.*
[p] *Radius's.*
[q] *Ax.* 8.

o XI. 2. Draw out C D to F, C E F ⊏ o C D
p Radius's. and (omitting the equals p C E, C D) E F ⊏
D F, to which adding the common F A, E F
q Ax. 8. ⊏ q (D F A ⊏ o D A. *Q. E. D.*

Theorem XLIII.

If one Circle touches another, a [...]
[b a] *drawn through both their Cen*[...]
[b f] *will fall upon the point of tou*[...]
ing [a].

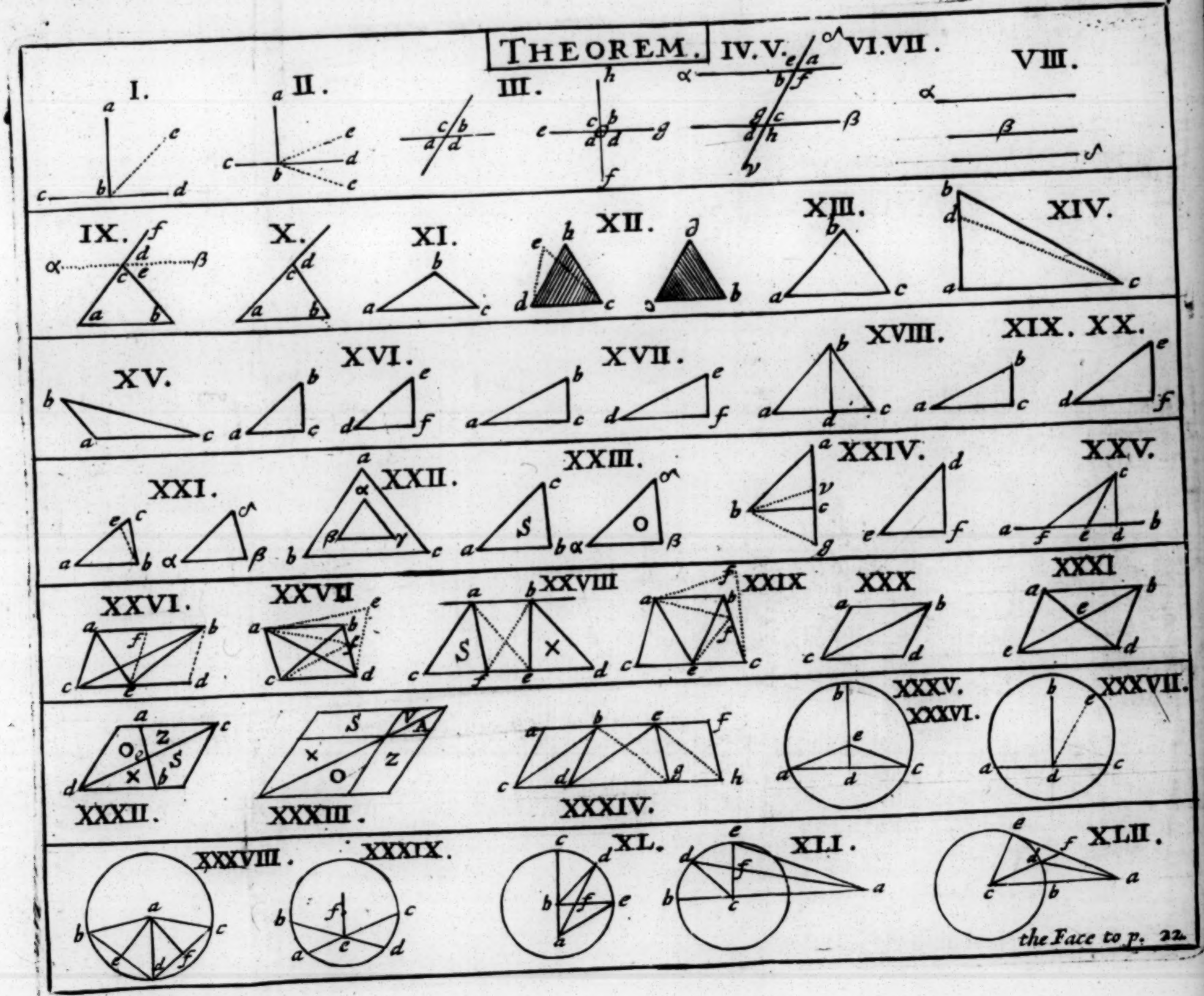
r Radius. IF not, let the Center of the lesser O be C,
s Ax. 8. the line that passes through both the Cen[...]
t XI. B, C, fall on D. Since then C is the center o[...]
u Radius's ⊐ O, C A = r C E, to which if you add
of the ⊏ the common B C, B C E (= s B C A) ⊏
O. A = u) B D. *Q. E. A.*

Theorem XLIV.

If Circles touch without, a line [b c] [...]
joins their centers will pass through [...]
point of touching [o.]

IF O be not the point of touching, let it b[...]
then B A, C A will be Radius's of the[...]
spective Circles; and by consequence = to[...]
w XI. O C ⊐ w B A, A C, which is a contradicti[...]

Theo[...]


THEOREM.
I.
II.
III.
IV. V.
VI. VII.
VIII.
IX.
X.
XI.
XII.
XIII.
XIV.
XV.
XVI.
XVII.
XVIII.
XIX. XX.
XXI.
XXII.
XXIII.
XXIV.
XXV.
XXVI.
XXVII
XXVIII
XXIX
XXX
XXXI
XXXII.
XXXIII.
XXXIV.
XXXV.
XXXVI.
XXXVII.
XXXVIII.
XXXIX.
XL.
XLI.
XLII.
the Face to p. 22.

XI
R
A

R
A
X
F
of
O

Circles

IF it be
point
Centers
ding the
E B,
Also
in O an
trary to
point.

One C

IF it be
there
to the C
as by X
the Cen
both C
to XL

Theorem XLV.

Circles whether within or without, touch but in one point [a].

IF it be poſſible, let them touch alſo in another point, as B, draw the Line E A through the Centers E F; therefore F A = [x] F B, and (adding the common E F) E F A (= [y] E F B) ⊏ [z] E B, againſt *def*. 21.

Alſo if they touch without (as in *fig*. XLIV) in O and A, then B A, A C = B O, O C, contrary to *XI*; wherefore they touch but in one point. *Q. E. D.*

[x] *Radius's.*
[y] *Ax.* 8.
[z] *XI.*

Theorem XLVI.

One Circle cuts another in two points only.

IF it be poſſible let them cut in four, A B C D, therefore four Lines drawn from theſe points to the Center of the O α, will be equal [a]. Whereas by *XLII*, thoſe will be ⊏ which are neareſt the Center of the O β. If the Centers fall within both Circles, it will be in like manner contrary to *XLI*.

[a] *Radius.*

Theorem

Theorem XLVII.

A perpendicular [b c] *to the end of Diameter* [b,] *falls all of it withou Circle.*

FRom the Center D, draw the Secant D C, Angle D B C is ⌊ [b], and thereupon ⊏ C B; therefore the subtendent D C ⊏ [d] a Radius, and by consequence the point C is w out the Circle. The same reason will hold fo points between C and B, therefore &c. *Q. E.*

[b] *Sup.*
[c] *X.*
[d] *XV.*

Theorem XLVIII.

The Angle [d a b] *of the Radius* [d *and Circumference is* ⊏ *than any a Angle* [d a e.]

DRaw D E perpendicular to A B; the An D E A is ⌊ [e], and ⊏ [f] D A E. The fore the subtend. D A ⊏ [g] D E, so that D E m fall within the O, and by consequence the An D A E is but a part of D A B, and so ⊐ tha *Q. E. D.*

[e] *Constr.*
[f] *X*
[g] *XV.*

Theor

Theorem XLIX.

The Tangent [ba] *of a Circle (def.27.) makes a* ∟ *with the Radius* [ca].

FOr if it made an acute, it would fall within the Circle ([h] since the Angle of the Circumference and Radius is ⊏ than any acute,) (*against* def.27.) if an obtuse, then the Angle on the other side C A D, would be acute; and by [h] consequence You'd fall within the Circle, against def. 27.

[h] *Pre.*

Theorem L.

Two Tangents [a b, a c,] *drawn from the same Point* [a], *are equal.*

FOr the Angles, D C A and D B A are ∟ [i], and thereupon [k] =. Also D C B = [l] D B C, (for D B [m] = D C,) which being taken from the two foresaid ∟ Angles; there [n] remains A B C = A C B, and by [o] consequence the subtendent A B = A C. *Q. E. D.*

[i] *Pre.*
[k] *Ax.* 2.
[l] *XIII.*
[m] *Radius.*
[n] *Ax.* 8.
[o] *XII.*

Theorem LI.

p Def. 33. *Lines,* [ad, bc] p *equidistant from* [the] *Center* [e] *are equal.*

Let fall the Perpendiculars EF, EG, from [the] Center E.

q Radius's. r Constr. s Ax. 2. t XXIV. u Ax. 7.

The side $\left\{\begin{array}{l} EA^{q} = EB \\ EG^{p} = EF \end{array}\right\}$ and the Angle EG[A] (∟ r) = s EFB. t Therefore the Tria[ngle] AEG = BEF, and the side AG = BF. [In] like manner the side GD = FC: therefore [the] whole AD u = FC.

Theorem LII.

Of Lines [ab, ac, ad,] *in a Circle,* [the] *greatest is the Diameter* [ad,] *the* [others] *are greater as nearer to this.*

w AD, AC, being Rad. and AE, common. x Ax. 5. y XX. z Pre.

1. AED (w = AEC) ⊏ AC. *Q.E.D.*
2. The Angle AEC ⊏ x AEB, th[ere]fore the subtend AC ⊏ y AB, (for the [other] sides AEC = AEB, all Radius's) *Q.E.D.*

If it be said that FG, is further from [the] Center than AC, and yet not ⊐ AC, from [the] point A draw a Line (as AB) equidistant fr[om] the Center with FG, which shall be = z to F[G] but ⊐ than AC, as before.

Theor[em]

Theorem LIII.

oppoſite Angles [bd, ac,], *of a four ſided Figure* [abdc] *inſcrib'd in a Circle, are equal to* 2∟.

aw the Radius's, LA, LB, LC, LD; the ſeveral Angles ſubtended by theſe Radius's will =, *viz.* [a] *XIII.*

$^{a}= \left\{ \begin{array}{c} G \\ H \\ M \\ N \end{array} \right\}$ Now all theſe Eight together, are = 4∟ ([b] becauſe the Figure ABCD, may be divided into two Triangles) therefore er half of them, EFIK, that is BD; or MN, that is, AC, are = 2∟. *Q. E. D.* [b] *X.*

Part II.

caſe the four ſided Figure inſcribed, (def. 90.) falls without the Center [i].

aw the Radius, IA, IC, IF, IH, to the ſeveral Angles. Now in the Triangle H, the Angle A[c] = H; and theſe Angles are =, *viz.* [c] *XIII.*

[c] *XIII.*
[d] *X.*

$$\left\{\begin{array}{c}A\\+\\B\\E\\F\end{array}\right\}^{c}=\left\{\begin{array}{c}C\\D\\G\\+\\H\end{array}\right.$$

Omitting therefore equals A and H, there main equal halvs, B E and DGH, which alt ther, are = 4 ∟ ([d] bec the whole Figure BC may be divided into 2 angles,) and therefore either half, (being opposite Angles,) are = to 2 ∟, *Q. E. D.*

Theorem LIV.

All Angles* [a, e] *in the same Segm* [b f d] (*def.* 30.) *are equal

[e] *Pre.*
[f] *Ax.* 8.

$C \begin{array}{l}+A= \\ C+E^{c}=\end{array}\Big\}\, 2$ ∟. Therefore (omittin common C) A[f] = E. *Q. E. D.*

Theorem LV.

An Angle [c f d] *in a Segment* [c e *the Center* [f], *is double to that* [c *which is at the Circumference.*

[g] *IX.*
[h] *XIII.*
[i] *Radius's.*

FOr the Angle CFD[g] = FDB + B; FDB[h] = B, (the subtend, FD being [i] = and B = A: Therefore CFD is double or A. *Q. E. D.*

Theo

Theorem LVI.

e Angle [a b c] *in the Semicircle is* $\llcorner$.

He Angles A, B, F + E, D, C [k] = 4 $\llcorner$. But F + D [l] = 2 $\llcorner$: Therefore A, B, E, C [m] = . But B, E, [n] = A, C, that is, either of ese pair are = $\llcorner$, and by consequence B + E, e Angle at the Semicircle is $\llcorner$. *Q. E. D.*

k *X.*
l *I.*
m *Ax. 8.*
n *XIII.*

Theorem LVII.

n Angle in a Segment $\left\{ \begin{matrix} \sqsubset \\ \sqsupset \end{matrix} \right\}$ *the Semicircle is* $\left\{ \begin{matrix} \textit{Acute,} \\ \textit{Obtuse.} \end{matrix} \right.$

Ecause CBD is [o] $\llcorner$, therefore CBA is [p] obtuse; and CBE, [p] acute. *Q. E. D.*

o *Pre.*
p *Def.* 8, 9.

Theorem LVIII.

qual Angles, [b, e; k, l;] *whether at the Center or Circumference, are subtended by equal Arches,* [a c, d f.] *of = Circles.*

Ecause the Angle E [q] = B, and the sides DEF [r] = ABC, therefore being laid upon one

q *Sup.*
r *Radius's of* = ○s.

one another, they shall [s] meet, and by co[n]quence the points D, F with the points A, likewise the ○s being [q] =, (if the Center be laid upon the Center B,) they shall me[et] and by consequence the Arch DF with A[C] wherefore they are =. *Q. E. D.*

[s] *Ax.* 4. [q] *Sup.*

Now K, L, are subtended by the same Arch which are already prov'd =.

Theorem LIX.

The Angle [a e f] *at the Center, stand[ing] upon half the Arch* [a f], *is = to* [the] *Angle at the Circumference, standing* [upon] *the whole Arch* [a f c].

FOr the Angle AEF [t] = CEF, (the Arch[es] being [u] = FC) but AEF is [w] double [to] ADF, and CEF is [w] double to CD[F]. Therefore ADF [x] = CDF, and both toge[ther] (*viz.* ADC) are = AEF; but ADC [y] [=] ABC, therefore [z] ABC = AEF. *Q. E.* [D.]

[t] *Pre.* [u] *Sup.* [w] *LV.* [x] *Ax.* 7. [y] *LIV.* [z] *Ax.* 5.

Theorem LX.

A Line [dc] *passing through a Circle at the Point* [c] *of touching, makes an Angle to the Tangent* [ab] = *to an Angle in the opposite Segment, viz.* $\underbrace{DCA = E}_{1}$, and $\underbrace{DCB = M}_{2}$.

DRaw the Diameter F C, and joyn F D.

1. Because the Angles [a] FDC, and [b] FCA, are $\llcorner$. Therefore K [c] + H, or DCA + H [d] = $\llcorner$, and (omitting the common H) there remains DCA [e] = (K [f] =) E. *Q. E. D.*

2. The Angle E + M [g] = (2 $\llcorner$) [h] = DCA + DCB: Therefore (taking away equals, DCA [i] and E,) there remains DCB = M. *Q. E. D.*

[a] *LVI.*
[b] *XLIX.*
[c] *X.* [d] *Ax. 9.*
[e] *Ax. 8.*
[f] *LIV.*
[g] *LIII.*
[h] *L.* [i] *I.*

CHAP. III. PART I.

Of Proportion in General.

Theorem LXI.

If four Terms (A . B :: C . D) *be Proportional, all the following* Changes *of the said terms shall be likewise Proportional.*

	A . B :: C . D	Let { A . B :: C . D 12 . 9 :: 8 . 6
1. *Inverted.*	B . A :: D . C	9 . 12 :: 6 . 8
2. *Alterned.*	A . C :: B . D	12 . 8 :: 9 . 6
3. *Compounded.*	A+B . B :: C+D . D	21 . 9 :: 14 . 6
Or.	A+C . C :: B+D . D	20 . 8 :: 15 . 6
4. *Divided.*	A−B . B :: C−D . D	3 . 9 :: 2 . 6
Or.	A−C . C :: B−D . D	4 . 8 :: 3 . 6
5. *Converted.*	A . A±B :: C . C±D	12 . 21 :: 8 . 14 12 . 3 :: 8 . 2
Or.	A . A±C :: B . B±D	12 . 20 :: 9 . 15 12 . 4 :: 9 . 3
6. *Mixed.*	A+B . A−B :: C+D . C−D	21 . 3 :: 14 . 2
Or.	A+C . A−C :: B+D . B−D	20 . 4 :: 15 . 3

That

That all these Changes are Proportional, may appear sufficiently plain from the numbers annext according to the following instances, in Theor. LXII. only Note by the way, that all these Changes of the Letters, agree also to the sides of the Triangle, (Fig 61.) which it would not be amiss to look upon all the way, as you read these over. *Note* 1.

2. The first Change, *Inverted*, affirms no more than this, that if A be as big in respect of B, as C is in respect of D, then B is as little in respect of A, as D is in respect of C; which may pass for an Axiom. 2.

Theorem LXII.

The second Change, Alterned $\begin{cases} A.C :: B.D \\ 12.8 :: 9.6. \end{cases}$

Suppose A, B, C, D; to be Lines divided into so many parts, *viz.* 12, 9, 8, 6; then 12. 8 :: 9. 6, for 8 is contained once and a $\frac{1}{2}$ (*viz.* 4.) in 12, and 6 once and a $\frac{1}{2}$ (*viz.* 3.) in 9. So that B and D are like parts of A and C, and by consequence A. C :: B. D. *Q.E.D.*

The same method will make all the rest of the Changes appear proportional.

A————
B—
C————
D—
a—.—.—.
b—.

But in order to a further Demonstration of this thing, (if it be required) let us consider, That the first Axiom about Proportion, which is naturally evident, and on which the whole Doctrine

[k] *Ax.* 10. Doctrine of it is founded, is this, [k] That eq
Fig. LXII. things (A, C,) have the same bigness in resp
of a third; (or of equals (B, D). Since in t
consideration equals are but the *same*, (for
Yards in a bundle, make but one Yard measur
In this case, (A . B :: C . D) Proportion
Equality is coincident; the Terms being b
Equal and Proportional.

Let us now increase the two first of th
equal terms, (A and C). And to do it by degr
(that the way may be clear,) let us first add
point to the measurer B, (as in *b*) (the effe
which is this; that A is not now so big in
[l] *Def.* 39. spect of *b*; as C is, in respect of D. $\left(\frac{A}{b}\right.$ [l] ⌐
and thus the Proportion is destroy'd: to rest
which) by consequence we must in *Proporti*
add 3 points to the measured A; (as in
[m] *Sup.* [m] because A is 3 times as big as B. Thus
Proportion is restored, (and the equality
stroyed) *a* being now three times as big as
just as C is to D. (*a* . *b* :: C . D). *Q. E. D.*

To Proceed.

It is naturally evident, in the first 4 ter
that A is as big in respect of C; as B is
respect of D, (because of their equality) (A
:: B . D) consider them now increast with
points: which we have already prov'd w
added *Proportionably*; (A being 3 times as
as B; and therefore ought to have 3 po
for the others one,) so that *a* and *b* are *Pro*
tionably increast above what they were befo

that is, *a* is as much bigger than it was (A) as *b* is than it was (B). Or (since A, C and B, D are equal) *a* is much bigger than C, as *b* is than D; *a* . C :: *b* . D, which is *Alterned.* *Q. E. D.*

And thus how many points soever (or Lines) you add to B, if you add of the same to A, in *Proportion* to its bigness above B, there will still be the same reason why *a* should be to C, as *b* to D. For since A and B were [n] Proportional to C and D; and are now Proportionally increas'd (in *a* and *b*) above what they were before, that is above C, D, it follows that they must still be Proportional to C, D; that is, *a* . C :: *b* . D. *Q. E. D.*

[n] *being equ.*

Theorem LXIII.

Like Parts are to one another, as their Wholes are.

B and D are like parts of the whole sides of the Triangle, it being prov'd in LXI. 3. that A+B . B :: C+D . D, 21 . 9 :: 14 . 6 therefore *Alternedly*, (LXI. 2.) it shall be, A+B . C+D :: B . C, 21 . 14 :: 9 . 6.

Fig. LXI.

Theorem

Theorem LXIV.

Reasonable Proportion.

1. *In Order.*

If in continued Proportion there be seve[ral] terms in two rows, A, B, C, ∺ D, E, &c. ∺ *Or if several terms be carried proportionaly, two in one row to two [in] another*, A . B :: D . E, and B . C :: E . F, &c. *then the extrea[ms] (A, C) in one row, shall be proporti[onal] to the extreams in the other.*

If,

A, B, C, *&c.* ∺ D, E, F, *&c.* ∺

Or if,

A . B		::	D . E		
12 . 6		::	18 . 9		
	B . C	::		E . F	
	6 . 4	::		9 . 6	
	&c.			*&c.*	

Then,

A . C	::	D . F	
12 . 4	::	18 . 6	

As may appear from the numbers, beca[use] $\frac{C}{4}$ is contain'd in $\frac{A}{12}$ three times, just as $\frac{F}{6}$, in $\frac{D}{18}$. But for further proof, $\frac{A}{C}$ may be ca[…]

whole; as being compoſed of $\frac{A}{B}$ and $\frac{B}{C}$. The like of $\frac{D}{F}$ as compos'd of $\frac{D}{E}$ and $\frac{E}{F}$. But the parts of both theſe wholes are $=$ (*viz.* $\frac{A}{B}{}^{\circ} = \frac{D}{E}$ and $\frac{B}{C}{}^{\circ} = \frac{E}{F}$) and therfore the wholes $\left(\frac{A}{C} \& \frac{D}{F}\right)$ muſt be $=$ too, (*Ax.* 5.) *Q. E. D.* ° *Sup.*

Theorem LXV.

If the former terms remaining, you prefix S *to the ſecond row, ſo that the two laſt* (B, C) *of the firſt row, may be proportional to the two firſt* (S,D) *of the ſecond row, it ſhall be as* A . C :: S . E. 2. *Diſturb'd*

$$\begin{array}{cccccccc} & & & & S. & & & \\ A, & B, & C, & \div\!\!\div & D, & E, & F, & \div\!\!\div \end{array}$$

Or if,

$$\begin{array}{ccccccc} & & & S. & & & \\ & & & 27 & & & \\ A & . & B & :: & D & . & E \\ 12 & . & 6 & :: & 18 & . & 9 \\ & & & :: & & & E \;.\; F \\ & & B \;.\; C & \Big\{ :: & S.D & & 9 \;.\; 6 \\ & & 6 \;.\; 4 & & 27.18 & & \end{array}$$

Then,

$$\begin{array}{ccccc} A \;. & C & :: S & . & E \\ 12 \;. & 4 & :: 27 & . & 9 \end{array}$$

For

p *Sup.* For, $\frac{E}{F}{}^{p}=\left(\frac{B}{C}{}^{p}=\right)\frac{S}{D}$. Therefore by Alte[r-]

q *Pre.* ning and Inverting $\frac{S}{E}=\left(\frac{D}{F}{}^{q}=\right)\frac{A}{C}$. *Q.E.D.*

It may appear also from the numbers.

Theorem LXVI.

Unproportional terms, are unproportion[al] in all the foregoing Changes.

Fig. LXII. If $\frac{A}{B}\sqsubset\sqsupset\frac{C}{D}$: Then also Alterned, &c. it shall [be] $\frac{A}{C}\sqsubset\sqsupset\frac{B}{D}$. For suppose the one point only adde[d] to B (as in *b*,) A is now not so big in respect of [b] as C is in respect of D (as we observed) $\left(\frac{A}{b}\sqsupset\frac{C}{D}\right)$ and by consequence *Alternedly* $\frac{A}{C}\sqsupset\frac{b}{D}$; for A i[s] but $=$ C, but *b* is $\sqsubset$ D, by a point. All which i[s] naturaly evident, by looking on the Lines, an[d] may easily be applied to the other Changes. T[o] give one instance in numbers, let it be $\frac{12.A}{9.B}\sqsupset\frac{C.}{D.}$ then Alterned, $\frac{12.A}{8.C}\sqsupset\frac{B.9}{D.5}$. For, 8 is con-tain'd but once and a $\frac{1}{2}$ (*viz.* 4.) in 12, whereas 5 is contain'd almost twice in 9: So that 9 i[s] bigger in respect of 5 than 12 is, in respect of 8, or, what is the same, 12 is $\sqsupset$ in respect of 8, &c.

CHAP.

CHAP. III. PART II.

f Proportion of Triangles, &c.

Theorem LXVII.

he sides [$\alpha\beta$, $\alpha\gamma$] of a Triangle are cut Proportionally by a Line [$\eta\theta$] Parallel to the base [$\beta\gamma$], viz. A . B :: C . D.

He Line $\delta\varepsilon$ being always mov'd Parallel towards $\beta\gamma$, will at last meet with it in points at once (for if any part of $\delta\varepsilon$ should uch before another, it would have been in-in'd in that part to $\beta\gamma$, and by consequence ould not be Parallel) and by consequence in e points β and γ. Since then $\delta\varepsilon$ begins at ce upon both the sides of the Triangle $\alpha\beta$, γ; and at the same time comes to the end of oth, β and γ, it follows, that when it is come o the middle of one, it is also at the middle f the other; when it has past over three parts f one, it has past the like of the other, &c. o that B, D, will be always *like* parts of the des A+B and C+D, and by consequence +B . B :: C+D . D, and by dissolving he composition, A . B :: C . D. *Q.E.D.*

Theorem

Theorem LXVIII.

A Line [$\eta\,\theta$] cutting the sides of a Triangle Proportionaly, is Parallel to the base [$\beta\gamma$].

FOr if any part of $\delta\,\epsilon$ (that is $\eta\,\theta$) should tend to the base $\beta\gamma$ before another, it would be inclin'd to the base in that part, and by consequence would not be Parallel. *Q. E. D.*

Theorem LXIX.

The base is to the Parallel Line, [$\beta\gamma$ ... as] the sides are to their parts next the top [α]: That is, E ┼ F . G :: B ┼ A . A :: D ┼ C . C.

FOr $\beta\,\delta$ being drawn Parallel to C D, it will be (by the Prec. *inverted*) D . C :: B . A

XXX. :: F . (E^r =) G therefore *compounded* (supposing C D the base, and η, the top) D ┼ C . C :: B ┼ A . A :: F ┼ G . G. *Q. E. D.*

Theorem LXX.

Parall. to the base, cuts off a part [X], *towards the top*, Like [::] *to the whole Triangle.* *Def.* 45.

Or the Angles $\left\{\begin{matrix}\alpha\gamma\beta^{f}=\alpha\theta n\\ \alpha\beta\gamma^{f}=\alpha n\theta\end{matrix}\right\}$ and α is [f] *IV.*
mmon, therefore X is equiangled to the whole
riangle.
Further, the side [t] DC . C :: FE . G :: BA . A: [t] *Pre.*
herefore *Alterned*, DC . FE :: C . G
and FE . BA :: G . A:
stly because B . A :: [u] D . C, therefore *Com-* [u] *LXVII.*
unded, B+A . A :: D+C . C, and *Alterned* *Inverted.*
+A . D+C :: A . C: Thus all the sides
out the equal Angles are proportional; and
[w] consequence X is :: to the whole Triangle. [w] *Def.* 45.
E. D.

Theorem LXXI.

Equiangled Triangles [$\alpha n\theta$, S.] *are* ::

Ut off, by the line $\beta\gamma$; $\alpha\beta, \alpha\gamma = \delta\varepsilon, \delta\zeta$, [x] *Sup.*
because the Angle α [x] $= \delta$, therefore X = [y] *XVII.*
, therefore the Angle γ [y] $=(\zeta$ [x] $=)\,\theta$, there- [z] *VI.*
re $\beta\gamma$ [z] Parall. $n\theta$; and by consequence the [a] *Pre.*
riangle $\alpha n\theta$, is [a] :: (X [b] ::) S. *Q. E. D.* [b] *Ax.* 10.

Theorem LXXII.

Triangles [$\alpha n \theta$, S.] are like, whose si[des] are Proportional.

c LXVII.
d Constr.
e Hyp. Alterned.
f Ax. 10.
g LXIX.
h XVI.
i Pre.
k LXX.

TAke $\alpha\gamma = \delta\zeta$, and draw $\gamma\beta$ Parall. [...]
Therefore $\frac{B+A}{A}$ [c] $= \left(\frac{D+C}{C^{d} \text{ or } \delta\zeta}\right)$ [e] $= \frac{B+}{\delta}$ [...]
therefore, A [f] $= \delta\varepsilon$: Further, $\frac{E}{F}$ [g] $= \left(\frac{D+}{C^{h} \text{or}}\right.$ [...]
[e] $= \frac{E}{\varepsilon\zeta}$: Therefore F [f] $= \varepsilon\zeta$; so then S [...]
equal sides to X, and by [h] consequence = A[...]
therefore is [i] :: (X [k] ::) $\alpha n \theta$. *Q. E. D.*

Theorem LXXIII.

Triangles [$\alpha n \theta$, S.] whose sides are [Pro]portional, are equiangled.

THis is Demonstrated in the operation o[f the] Precedent; for the Supposition b[e]in[g the] same in both, it is there prov'd, that S ha[s =] Angles to (X, which has [l] = Angles to) [...]
Q. E. D.

l LXX. and Def. 45.

Theo[rem]

Theorem LXXIV.

Triangles [$\alpha\eta\theta$, S.] *are* ::, *which have one Angle* =, [$\alpha = \delta$] *and the ſides about that Angle Proportional* B-|-A . C-|-D :: $\delta\varepsilon . \delta\zeta$.

CUt off, (by the Line $\beta\gamma$) $\alpha\beta$, $\alpha\gamma = \delta\varepsilon$, $\delta\zeta$. Becauſe the Angle α[m] = δ, therefore X[n] = S, and by [o] conſequence is ::, now becauſe A-|-B . ($\delta\varepsilon$[p] or) A[m] :: C-|-D . ($\delta\zeta$[p] or) C : Therefore [q] $\beta\gamma$ Parallel $\eta\theta$, and by conſequence $\alpha\eta\theta$ is [r] :: (X ::) S. *Q. E. D.*

[m] *Sup.*
[n] *XVII.*
[o] *LXXI.*
[q] *Conſt.*
[q] *LXVIII.*
[r] *LXVII.*

Theorem LXXV.

Triangles [$\alpha\eta\theta$, S.] *are* ::, *which have two ſides Proportional* [B-|-A . D-| C :: $\delta\varepsilon . \delta\zeta$], *and the Angle oppoſite to theſe ſides*, = [$\theta = \zeta$] *and another of the ſame kind.*

CUt off (by the Line $\beta\gamma$) $\alpha\beta$, $\alpha\gamma = \delta\varepsilon$, $\delta\zeta$. Becauſe $\frac{B \text{-|-} A}{(\delta\zeta^{ſ}\text{or})A}$ [ſ] = $\frac{D \text{-|-} C}{(\delta\varepsilon^{t}\text{or})C}$: [u] Therefore $\beta\gamma$ Parallel $\eta\theta$. Now the Angle ζ[ſ] = (θ[w] =) γ: Therefore X[x] = S, and by conſequence [y] ::, but $\alpha\eta\theta$ [z] :: (X ::) S. *Q. E. D.*

[ſ] *Sup. Alterned.*
[t] *Conſtr.*
[u] *LXVIII.*
[w] *VI.*
[x] *XXIV.*
[y] *LXXI.*
[z] *LXVII.*

Theorem LXXVI.

A Right-angled Triangle [a b c], *divided by a Perpendicular* [a d] *from the* ⌊ *to the subtendent*, *is* :: *its parts* [a b d, a d c].

FOr each part has one Angle common to the whole, *viz.* B and C, and one Angle ⌊ (*viz.* at D,) [a] = B A C: Therefore the whole is [b] equiangled to its parts, and by consequence [c] is :: to them. *Q. E. D.*

[a] *Ax.* 2.
[b] *XXII.*
[c] *LXXI.*

Thorem LXXVII.

The sides of a Triangle are Proportional t the parts of its base, divided by a Lin that cuts the opposite Angle in the midd f g. A :: D. C.

DRaw out A, Infinitly, and G I Parallel I H therefore the Angle F G I [d] = (H F G [e] = H F E [d] =) G I F, and therefore the subtenden F I [f] = F G. [g] But, (B that is) F G. A :: D. C *Q. E. D.*

[d] *IV.*
[e] *Sup.*
[f] *XII.*
[g] *LXVII.*

Theorem

Theorem LXXVIII.

Triangles of the same height, (or between the same Parallels) are to one another as their bases, that is, S . Z :: DC . CF.

TAke DB = CF, therefore X[h] = Z. Now if CA be mov'd, on the point A, as on a Center, towards AB, they will at last meet: and CA shall at the same time have past over the whole Triangle, (ABC,) and the base (BC;) and by consequence when it has come to the middle of one, it will be at the middle of the other; and so, in Proportion, whatsoever part it has past over of the Triangle, it will have past over a like of the base; and (by conseq.) will divide the Triangle and base Proportionally; that is, that S shall be to (X =) Z :: DC.(DB =)CF. *Q. E. D.*

[h] *XXVI.*

Theorem LXXIX.

Equal Triangles X, S, *having one = Ang.* [*at* C] *have their sides about this Ang. reciprocally proportional,* AC . CE :: DC . CB.

LEt the = Angles ECD, BCA, be so joyned that EA, BD may be Right-lines, (which is possible;

poſsible ; as may eaſily appear from III. ; the II. does from I. Then joyn BE: Now

[i] *Pre.* [k] *For* X=S *Sup.*

$\frac{AC}{CE}$ [i] $=\left(\frac{X}{Z}\right.$ [k] $= \frac{S}{Z}$ (*Ax.* 10.) [i] $\left.=\right)\frac{DC}{CB}$. *Q. E. D.*

Theorem LXXX.

Parallelograms [O, S,] *of the ſame heig[ht] are to one another, as their baſes* [c[g], dg].

[l] *XXX.* [m] *LXXVIII* [n] *Ax.* 7.

FOr the [l] Triangle CD, is $\frac{1}{2}$ O; alſo DF[G] is $\frac{1}{2}$ S. But $\frac{CD}{\frac{1}{2}O}$ [m] $=\frac{DG}{\frac{1}{2}S}$, and *Alter[nately]* $\frac{CD}{DG}=\left(\frac{\frac{1}{2}O}{\frac{1}{2}S}\right.$ [n] $\left.=\frac{\text{whole}}{\text{whole}}\right)\frac{O}{S}$. *Q. E. D.*

Theorem LXXXI.

Parallelograms [O, S,] *having one An[gle]* =, [*at*, c] *have their ſides about t[he] Angle reciprocally proportional*, E C . CA :: BC . CD.

[o] *Sup.* [p] *LXXII.* [q] *LXXIX.*

FOr ſince S [o] = O, therefore $\frac{1}{2}$ S (*viz.* [the] Triangle ECD) [p] = $\frac{1}{2}$ O, (*viz.* BCA) and [q] therefore, E C . C A :: B C . C[D]. *Q. E. D.*

Theorem

Theorem LXXXII.

grams [S, O,] *that have one Ang. and by consequence all Rectangles) one another in a reason* [r] *compoun- that of their sides.* [r] *Def.* 43.

e such Figures have both Length and th, to know the *reason* of one to the t is necessary to compare both their nd Breadth, (that is, their sides) and e *reason* of both these, is *compounded* of the Figures. For instance, if S as long, and thrice as broad as O, two times thrice, (that is, 6 times) O, (as may be seen by the prickt S,) so that $\frac{S}{O} = \frac{DB}{BA} + \frac{EB}{BC}$. *Q. E. D.*

Theorem LXXXIII.

arallelograms [S, O,] *are in a icate* reason *to that of their Ho- gous sides*, DB, BA, EB, BC.

eason of Pgrs. as we have said, is *pounded* or,) made up of the *reasons* of ngth and Breadth. But now in *Like* e *reasons* of their Length and Breadth,

poſſible ; as may eaſily appear fror
[i] *Pre.* the II. does from I. Then joyn BE
[k] *For* X=S *Sup.* $\frac{AC}{CE}\ {}^{i}=\left(\frac{X}{Z}\ {}^{k}= \frac{S}{Z(Ax.10.)}\ {}^{i}=\right)\frac{DC}{CB}$. *Q.*

Theorem LXXX.

Parallelograms [O, S,] *of the ſam are to one another, as their baſ* dg].

[l] *XXX.* FOr the [l] Triangle CD, is $\frac{1}{2}$O; al
[m] *LXXVIII* is $\frac{1}{2}$ S. But $\frac{CD}{\frac{1}{2}O}\ {}^{m}= \frac{DG}{\frac{1}{2}S}$, and
[n] *Ax.* 7. $\frac{CD}{DG}=\left(\frac{\frac{1}{2}O}{\frac{1}{2}S}\ {}^{n}=\frac{\text{whole}}{\text{whole}}\right)\frac{O}{S}$. *Q. E.*.

Theorem LXXXI.

Parallelograms [O, S,] *having o* =, [*at*, c] *have their ſides* *Angle reciprocally proportional* CA :: BC . CD.

[o] *Sup.* FOr ſince S [o] = O, therefore $\frac{1}{2}$ S
[p] *LXXII.* Triangle ECD) [p] = $\frac{1}{2}$ O, (*viz*
[q] *LXXIX.* and [q] therefore, EC . CA :: B
Q. E. D.

Theorem LXXXII.

Parallelograms [S, O,] *that have one Ang.* =, (*and by consequence all Rectangles*) *are to one another in a reason* [r] *compounded of that of their sides.* [r] *Def.* 43.

FOr since such Figures have both Length and Breadth, to know the *reason* of one to the other, it is necessary to compare both their Length and Breadth, (that is, their sides) and out of the *reason* of both these, is *compounded* the *reason* of the Figures. For instance, if S be twice as long, and thrice as broad as O, then S is two times thrice, (that is, 6 times) as big as O, (as may be seen by the prickt Lines, in S,) so that $\frac{S}{O} = \frac{DB}{BA} + \frac{EB}{BC}$. *Q. E. D.*

Theorem LXXXIII.

Like *Parallelograms* [S, O,] *are in a Duplicate* reason *to that of their Homologous sides*, DB, BA, EB, BC.

THe reason of Pgrs. as we have said, is (*compounded* or,) made up of the *reasons* of their Length and Breadth. But now in *Like* Pgrs. the *reasons* of their Length and Breadth,

 are

are the ſame, (that is, S is juſt as much broader as it is longer, than O:) So that, whereas in the former Propoſition, to find the reaſon of S to O, we were to joyn together the reaſons of their Length and Breadth; here it will be ſufficient to take the *reaſon* of either, twice because they are both the ſame: Thus, if S be twice as long, and (by conſequence) twice as broad as O, then it is two times twice (that is 4 times) as big as O, and this is call'd *Dupli-*

[s] *Sup.* *cate* reaſon. For Lines thus, $\frac{DB}{EB}$ [s] $= \frac{BA}{BC}$, and

[t] *Pre.* *Alterned*, $\frac{DB}{BA} = \frac{EB}{BC}$: But $\frac{S}{O}$ [t] $= \left(\frac{DB}{BA} + \frac{EB}{BC} = \right.$

$2\,\frac{DB}{BA}$ or, $2\,\frac{EB}{BC}$. *Q. E. D.* For further Illuſtration,

Fig. II. If the Length of S, be to the Length of O as 4 to 3, by [u] conſequence the Breadth must

[u] *Def.* 42.

[w] *Pre.* be as 4 to 3 alſo. Then $\frac{S}{O}$ [w] $= \frac{4}{3} + \frac{4}{3}$, that is, $= 2\,\frac{4}{3}$: *viz.* $\frac{16}{9}$, as appears by the little Squares in S and O.

Theorem LXXXIV.

Parallelograms* [S, O,] *are equal, which have one Angle* =, [α, β,] *and the ſides about this, reciprocally proportional.

Viz. If A . D :: C . B, then S = O. Now *Alterned*, A . C :: D . B; that is, S is as much

much longer than O; as O is broader than S, and [x] since the bigness of S to O, is made up of the bigness of the sides, if S be 3 times longer than O; and O 3 times broader than S, these two reasons in being *compounded* destroy one another, and S is left = O. *Q. E. D.* [x] *LXXXII.*

Theorem LXXXV.

All Squares are :: *(Figures, or) Pgrs.*

FOr A [y] = B and C [y] = D, therefore, A . C :: [z] B . D, and *Alterned*; A . B :: C . D: Therefore S is [a] :: O. *Q. E. D.*

[y] *Sup.*
[z] *Ax.* 10.
[a] *Def.* 45.

Theorem LXXXVI.

Like Rect-angles [S, O,] *are to one another as the Squares of their Homologous sides*, A . B.

FOr $\frac{S}{O}$ [b] $= \left(2\,\frac{A}{B} \right.$ [c] $= \left. \right) \frac{Aq}{Bq}$. *Q. E. D.*

[b] *LXXXIII*
[c] *Pre. with LXXXIII.*

Theorem

Theorem LXXXVII.

F.LXXXII *Triangles that have one Angle* $=$ *, are* [to] *one another in a reason compounded* [of] *that of their sides.*

d XXX. FOr Triangles are [d] $\frac{1}{2}$ Pgrs. (Diameters bei[ng] drawn from E to D, and from A to C,) a[nd] likewise receive the same sides with S and [O].

e LXIII. *f LXXXII.* But $\frac{\frac{1}{2}S}{\frac{1}{2}O}{}^{e}=\left(\frac{S}{O}{}^{f}=\right)\frac{DB}{BA}+\frac{EB}{BC}$. *Q. E. D.*

Theorem LXXXVIII.

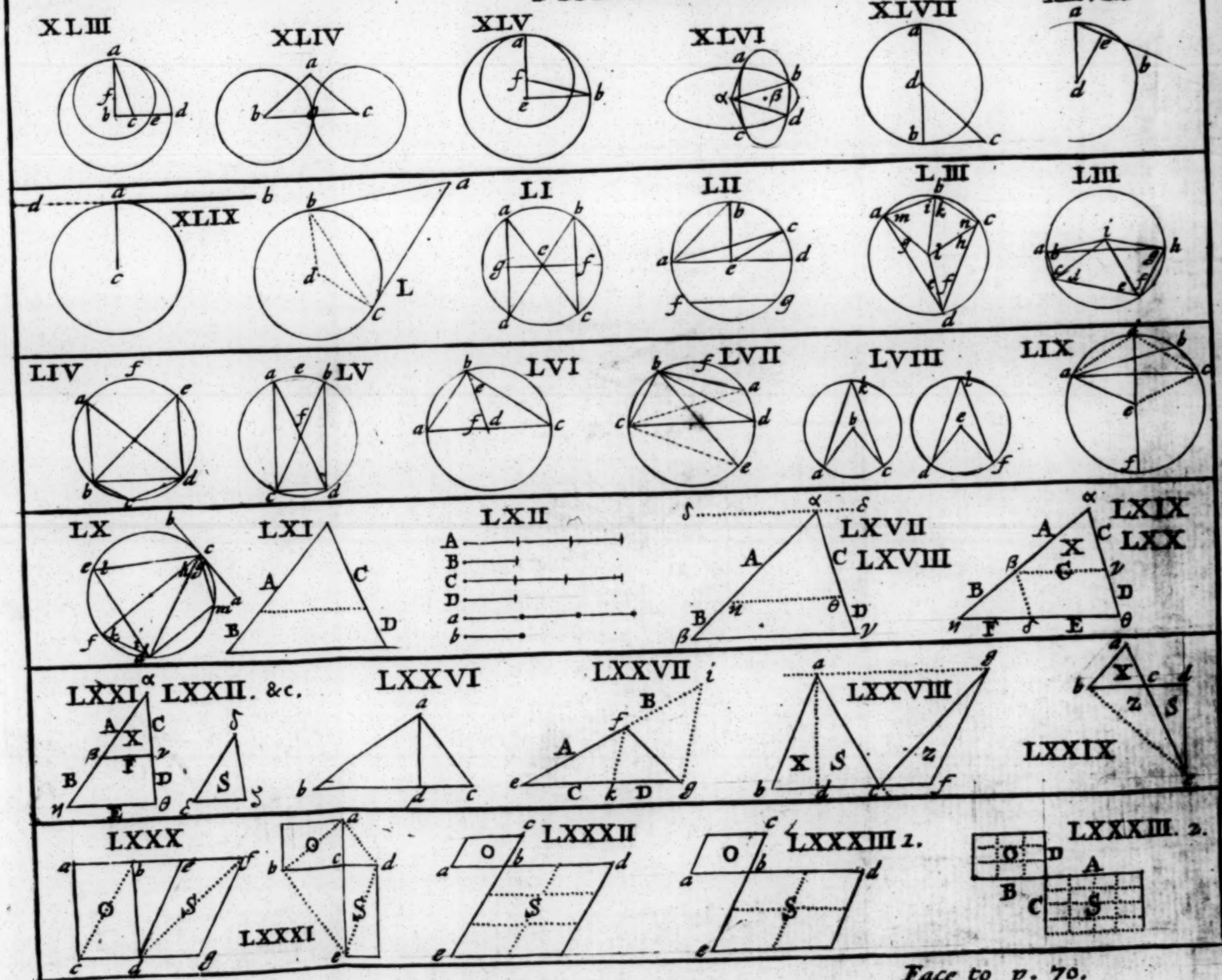
f.LXXXIII Like *Triangles, are in a Duplicate rea[son]* *to that of their Homologous sides.*

FOr these, again, are $\frac{1}{2}$ the Pgrs. S and [O].

g LXIII. *h LXXXIII.* Therefore $\frac{\frac{1}{2}S}{\frac{1}{2}O}{}^{g}=\left(\frac{S}{O}{}^{h}=\right)2\frac{DB}{BA}$ or $2\frac{E[B]}{B[C]}$ *Q. E. D.*

Theore[m]

THEOREMS.



Face to p. 70.

F.L.

d X

e L.
f L.

f.L.

g L
h L

Of P

ke Pgr
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Theorem LXXXIX.

ke Pgr's. [a d, e g] *plac'd* ::, *and having one common Angle* [c], *receive the same Diameter* [c b].

f not, let the inner Pgr. be CFHG; and draw F G parallel D B: Therefore, the ngle C G F[i] = (D[k] =) C G H, (for C E H G supposed :: to A D.) The like, if H fall on e other side the Diameter, therefore since H n fall on neither side the Diameter, it must ll upon it. *Q. E. D.*

[i] *IV.*
[k] *Def.*

Theorem XC.

Like Figures [abcde, fghik,] *of many sides, may be divided into an equal number, of :: Triangles, by the Lines* [a c, a d, f h, f i].

THe first part is plain only by looking on the Figures; the second, that the Triangles are ::, is prov'd thus: The Angle B = G; and the sides, A B . B C :: [l] F G . G H: Therefore the Triangle A B C[m] like F G H; and by the same reason A E D like F K I. Lastly C A D n like H F I; (for the whole Figures were [l] equiangled, and equal Angles have been taken away on[o] each side, therefore there remains = Angs. to CAD and HFI. *Q.E.D.* Theorem

[l] *Sup.*
[m] *LXXIV.*
[n] *LXXI.*
[o] *Being prov'd* ::

Theorem XCI.

Like *Figures* [abcde, fghik,] a*r*[e to] *one another, in a* Duplicate *reason* [(or] *as the Squares) of their Homologous* s[ides.]

[p] *XXX.* [q] *LXXXIII.* [r] *LXXXV.* [s] *Pre.*

FOr (every Triangle being [p] $\frac{1}{2}$ a Pgr.) the Triangles $\frac{ABC}{FGH}$ [q] $= 2\,\frac{AB}{FG}$ ([r] $= \frac{ABq}{FGq}$) … $\frac{AED}{FKI} = 2\,\frac{AE}{FK}$ and $\frac{CAD}{HFI} = 2\,\frac{CD}{HI}$. But n[ow] because AB . BC :: FG . GH / BC . CD :: GH . HI } th[ere]-

[t] *LXIV.*

fore [t] AB . CD :: FG . HI; and by the sa[me] reason AE . CD :: FK . HI; and *Altern*[ately] both together $\frac{AB}{FG} = \frac{CD}{FI} = \frac{AE}{FK}$: Since t[hen] the reason of all these Homologous sides, is eq[ual,]

[u] *Ax.* 10.

the Triangles $\frac{ABC}{FGH}$ or $\frac{AED}{FKI}$ or $\frac{CAD}{HFG}$ [u] $= 2\,\frac{A}{F}$…

[w] *LXIII.*

Therefore the wholes $\frac{ABCDE}{FGHIK}$ [w] $= 2\,\frac{AB}{FG}$ … $\frac{ABq}{FGq}$. *Q. E. D.*

Theor[em …]

Theorem XCII.

[...]cles are to one another in a Duplicate *[...]ason of their Diameters, (or, as the Squares of Diameters* LXXXVI. *with* LXXXIII.)

[...]Hat is $\frac{\bigcirc, ADF}{\bigcirc, \alpha\delta\varphi} = 2\frac{AB}{\alpha\beta} = \frac{A\,Bq}{\alpha\beta q}$. If not, $\frac{\bigcirc, ADF}{\bigcirc, \alpha\delta\varphi} \sqsubset 2\frac{AB}{\alpha\beta}$. And in the ○, ADF, [...] there be in inscrib'd a Polygon (or many [...]d Figure) Z in such manner that $\frac{Z}{\bigcirc, \alpha\delta\varphi}$ [...] $2\frac{AB}{\alpha\beta}$ (which appears to be possible, because [...] sides of the Polygon may be so multiplied [...] they come to be = Circumference of the [...] it self, and by consequence whatsoever the [...] exceeds, Z may likewise exceed.) Then [...]he ○, $\alpha\beta\varphi$, let the Polygon X be * inscrib'd, [...]. Now the △, AFB is [x] :: $\alpha\varphi\beta$; and by [...]sequence $\frac{AFB}{\alpha\varphi\beta} = {}^{y}\, 2\frac{AB}{\alpha\beta}$ And. $\frac{Z}{X}{}^{z} = \left(\frac{AFB}{\alpha\varphi\beta}\right) 2\frac{AB}{\alpha\beta}$: But $\frac{Z}{\bigcirc, \alpha\delta\beta}{}^{a} \sqsubset 2\frac{AB}{\alpha\beta}$. [b] There[...]e X $\sqsubset$ ○, $\alpha\delta\beta$. [c] *Q. E. A.* Therefore $\frac{ADF}{\alpha\delta\varphi} = 2\frac{AB}{\alpha\beta}$ or $\frac{ABq}{\alpha\beta q}$. *Q. E. D.*

* *See Problem.*
[x] *XC.*
[y] *Pre.*
[z] *LXXIII.*
[a] *Constr.*
[b] *Ax.* 11.
[c] *Ax.* 5.

Theorem

Theorem XCIII.

All the external Angles of any Polyg[on] equal 2 ∟ Angles.

1. A Polygon may be divided into so[many] Triangles, as it has sides; by [lines] drawn from any point within, to the se[veral] Angles; this appears by looking on the Fi[gure].

2. And by consequence it contains tw[ice as]
d X. many [d] ∟ Angles, (as it has sides;) exce[pt 4,]
e I. which are about the point in the middle, [for] all the Angles about a point are $=$ 4 ∟.

3. Then, drawing out the several [sides,]
f XCIII, 2. Each against the other makes 2 Angles [=] ∟; and by consequence all the Angles tog[ether] inward and outward, will be $=$ to tw[ice as] many ∟, as there are sides.

4. But the inward Angles are prov'd [=] these, except 4, [f] and therefore the out[ward] Angles remain $=$ 4 ∟. *Q. E. D.*

Theorem XCIV.

Three only Regular *Figures (that is, w[hose] sides and Angles are $=$) can fill a sp[ace,] viz. Triangles, as in* A, *Squares, [as in]* C, *and Hexagons, as in* B.

FOr since all the Angles about a point
g I. [g] $=$ 4 ∟.

1. The Angle of a *regular* △ is [h] $= \frac{1}{3}\, 2$ ⌊ (because all the 3 Angles are $=$) and by consequence 6 of these joyn'd together, as in A, are $= 4$ ⌊ Angles, for $\frac{6}{3}\, 2$ ⌊ $= 4$ ⌊. *h X.*

2. The Angle of the □ is [i] ⌊; and by consequence the 4 at C are $= 4$ ⌊. *i Def.*

3. The Angle of a Hexagon is [k] $= 1$ and $\frac{1}{3}$ ⌊, and by consequence the 3 joyn'd together in B are $= 3$ and $\frac{3}{3}$ ⌊, that is, 4 ⌊. *k Pre. 2.*

Now all Figures that have more sides than this, have their Angles bigger than it, and by consequence 3 of them (which is the [g] least number) that can make a plane Angle) will be $\sqsupset$ 4 ⌊; In the Pentagon (of 5 sides) 3 Angles will be ⊐, 4 ⊏ than [k] 4 ⌊. *Q. E. D.*

Theorem XCV.

Like Figures, [S, O, Z, X,] *fram'd a like upon Proportional bases,* [a . b :: c . d] *will be themselves also Proportional,* viz. S . O :: Z . X.

FOr, $\frac{S}{O}\,{}^{l}\!=\!\left(\frac{A}{B}\,2\overset{m}{=}\,2\frac{C}{D}=\right)\frac{Z}{X}$. *Q. E. D.*

l XCI.
m LXIII, for A . B :: C . D, Sup.

Theorem XCVI.

In a Circle, [b a c, b a d,] *the Angles are Proportional to the Arches* [b c, b d,] *that subtend them.*

FOr the Line A B, being mov'd towards C, at the same time will dispatch both the

A, and the Arch BC; and by conſequen
when it comes to the middle of one, it will
at the middle of the other, and ſo forwar
ſo that in fine, whatſoever part BAD will
of BAC, BD will be the ſame part of B
Therefore the Angle BAD, will be to B
:: Arch B, to BC. *Q.E.D.*

Theorem XCVII.

A Polygon conſcrib'd about a Circle is to a Right Angled Triangle, whoſe b [ββ] is the Compaſs of the Polygon, its height [αβ] the Radius of the Cir

DRaw the Radius's AH, AI, &c. to the
of the Polygon at the point of touchi
then take βH = BH; and HC = HC;
ſo forward, 'till you come to NB; then dr
αZ parallel ββ. Erect the Perpendicul
HO, IP, KQ, &c. and joyn Oβ, OC; P
[n] *Conſtr.* PD, &c. Now, becauſe the Angle OHB
[o] *Sup.* (∟ [o] =) αβH, therefore HO is [p] parallel
[p] *IV.* αβ; and by [q] conſequence HO = (αβ [o] =
[q] *XXX.* HA; therefore the △s, BAC and βOC ha
[r] *XXVIII.* the ſame height; therefore BAC [r] = (βO
[s] *XXVI.* [s] =) βAC: In like manner CαD = (C
=) CAD, &c. and, in fine, Gαβ = (GZB
GAB. So then all the parts being equal,
[t] *Ax. 3.* whole △αββ [t] = to the Polygon. *Q.E.D.*

Theorem XCVIII.

Everry regular *Polygon is equal to a Right Angl'd* $\triangle$, *whoſe baſe is the Compaſs of the Polygon, and its height, a Perpendicular* [a h] *to the ſide of the Polygon* [b c] *from the Center.*

FOr that Perpendicular *(a h)* will be the ſame with the Radius of the Circle that may be inſcribed in ſuch a Polygon, and than the reſt follows from the Precedent.

Theorem XCIX.

A Circle is $=$ *to a* $\llcorner$ *Angled* $\triangle$, *whoſe baſe is the Circumference of the Circle; and its height, the Radius of it.*

1. EVery [u] Polygon conſcrib'd is $\sqsubset$, and inſcrib'd is $\sqsupset$, then the $\bigcirc$. [u] *Ax.* 5.
2. The compaſs of a Polygon conſcrib'd, is $\sqsubset$, and inſcrib'd is $\sqsupset$, than the [u] circumference of the Circle.
3. This $\llcorner$ Angled $\triangle$ will be [w] $\sqsupset$ than any Polygon conſcrib'd, and $\sqsubset$ than any inſcrib'd, (becauſe the circumference of the $\bigcirc$, which is the baſe of this Triangle is [x] $\sqsubset$ than the compaſs of any inſcrib'd.) Therefore it will be $=$ to the $\bigcirc$. [w] *XCVII.* [x] *Ax.* 5.

For if this Triangle be $\sqsubset$ than any thing that is bigger than the ○, and $\sqsupset$ than any thing that is $\sqsupset$ than the ○, it follows that it muſt be equal to the ○. *Q. E. D.*

Note. And this is call'd the Squaring of a Circle, *viz.* to find a Right-lined figure equal to a Circle upon this ſuppoſition, that the baſis given is equal to the circumference of the Circle. But actually to find a Right-line $=$ to the circumf of a ○, is not yet diſcover'd Geometrically.

Theorem C.

An equal Legg'd Triangle [abc] *in* *Segment, is* $\sqsubset$ *than* $\frac{1}{2}$ *the Segment.*

DRaw the Radius F B Perpend. A C, and the Tangent D B E, which will be Parallel to A C, (becauſe the Angle A G B [y] is ∟ [z] = D B G) further draw A D and C E Parallel G B; D C will be a Pgr. and the Triangle ABC half of it; but the whole Pgr. is $\sqsubset$ than the Segment A B C; therefore $\frac{1}{2}$ Pgr. *viz.* ABC $\sqsubset$ [a] $\frac{1}{2}$ the Segment. *Q. E. D.*

[y] *Conſt.*
[z] *XLIX.*
[a] *LXIII.*

Theorem

Theorem CI.

The Triangle DCB *is* ⊏ *than* ½ *the mixt Triangle* ACB.

LEt AB and DC be Tangents: Therefore DC = [b] DA, and Angle (DCF, [c] is ∟ [d] = DCB; therefore the subtend. DB [e] ⊏ (DC =) DA: Therefore the Triangle DCB [f] ⊏ (DCA [g] ⊏) ADC, the Triangle mixt, and by consequence is more than ½ ABC the Triangle mixt. *Q. E. D.*

[b] *L.*
[c] *XLIX.*
[d] *I.*
[e] *XV.*
[f] *LXXVIII*
[g] *Ax.* 5.

Note. From the two Preceding Propositions it appears, that figures inscrib'd and conscrib'd, if the number of their sides be doubled, do approach the Circle by more than half the space that was before left.

Theorem CII.

A Circle contains more space, than any other figure that is equal to a Circle.

FOr a ○ is = [h] ∟ Angled △ whose base is the circumference of the ○, and height, the Radius AB. But the Square CD (which we suppose = to the ○) is [i] = to ∟ Angl'd △, whose base indeed, is the same with the former, (because the compass of the Square is supposed = circumference of the ○:) but the height is the Perpendicular AE. *Q. E. D.*

[h] *XCIX.*
[i] *XCVIII.*

F 2 CHAP.

CHAP. IV. PART I.

Of the Power of Lines.

The Power *of Right-lines, is when they are so plac'd as to comprehend the greatest space; which is when they are set at Right angles. Therefore a* ▭ *and* □ *are call[ed] the* Power *of Lines.* Note, *two letters se[t] together* AB *or* AE, *(or with a Note of Mu[l]tiplication between them*, A×B, A×E,) *signifie a* ▭ *made of those two Lines. An[d]* AA, *or* A×A, *or* Aq, *or* QA, *signifie the Square of the Line* A.

Theorem CIII.

A Rectangle [ZB] *of two whole Lin[es]* [ZB] *is = to the Rectangles* [AB+EB] *that are made of one whole* [B] *and of th[e] parts* [A, E] *of the other.*

IF $Z = A + E$ then $ZB = AB + EB$ } For to = bases (viz. Z, and A+E) there added an = height (B) [k] therefore the Rec[t]angles are =. *Q. E. D.*

k *XXXIV.*

Scholium

Scholium.

angle of the whole Lines [Z B] *is* *Fig. II.*
to the Rectangles of the parts of
ine. [*viz.* C, D. A, E.]

$$\left.\begin{matrix} =\left\{\begin{matrix} AB = \\ + \\ EB = \end{matrix}\right. & \overset{1}{\left.\begin{matrix} AC + AD \\ \\ EC + ED \end{matrix}\right\}} \end{matrix}\right.$$ [m] there- [l] *Pre.* [m] *Ax. 6.*

: AC + AD + EC + ED. *Q.E.D.*

Theorem CIV.

gle [Z A] *of the whole, with one*
rts, [A] *is* = *to the* Q *of the*
rt, [A] + *to the* ▭ *of the*
[A, E] *together.*

A [n] = AA (*i.e.* Aq) + Æ. For the [n] *CIII.*
:ight, A is added to [o] = bases Z and [o] *Ax. 5.*
:refore the ▭ Z A is = AA + AE.

CHAP. IV. Par

Of the Power of Lir

The Power *of Right-lines, is are so plac'd as to comprehend th space; which is when they are se angles. Therefore a* ▭ *and* □ *the* Power *of Lines.* Note, *two together* AB *or* AE, *(or with a* N *tiplication between them*, A × B *signifie a* ▭ *made of those two Li* A A, *or* A × A , *or* A q, *or* QA *the Square of the Line* A.

Theorem CIII.

A Rectangle [Z B] *of two wi* [ZB] *is = to the Rectangles* [*A that are made of one whole* [B *parts* [A, E] *of the other.*

IF Z = A + E } For to = I then ZB = AB + EB } Z, and A +
k XXXIV. added an = height (B) k therefor angles are =. *Q. E. D.*

Scholium.

The Rectangle of the whole Lines [Z B] *is also* = *to the Rectangles of the parts of each Line.* [*viz.* C, D. A, E.] *Fig. II.*

FOr, ZB[l] = $\left\{ \begin{array}{l} AB = \\ + \\ EB = \end{array} \right. \left. \begin{array}{l} AC + AD \\ \\ EC + ED \end{array} \right\}$ [m] therefore, ZB = AC + AD + EC + ED. *Q.E.D.*

[l] *Pre.*
[m] *Ax.* 6.

Theorem CIV.

A Rectangle [Z A] *of the whole, with one of its parts,* [A] *is* = *to the* Q *of the same part,* [A] + *to the* □ *of the Parts* [A, E] *together.*

FOr, Z A[n] = A A (*i.e.* Aq) + Æ. For the same height, A is added to[o] = bases Z and A, E: Therefore the □ Z A is = A A + A E. *Q.E.D.*

[n] *CIII.*
[o] *Ax.* 5.

Theorem CV.

The Q of the whole is = Q of the parts, + to the Rectangles made of the same parts, viz. Zq = Aq + Eq + 2 Æ.

p CIII. q Pre.

For $ZZ^{p} = \begin{cases} ZE^{q} = \\ + \\ ZA^{q} = \end{cases} \begin{matrix} EE\ (i.e.\ Eq) + EA \\ \\ AA\ (i.e.\ Aq) + AE \end{matrix}$

that is, ZZ = Eq + Aq + 2 Æ. *Q. E. D.*

Theorem CVI.

Fig. Preced.

$$Zq + Eq = 2\,ZE + Aq.$$

r CIII. s CIV.

For $Zq^{r} = \begin{cases} ZE \quad ^{s}= \\ + \\ (ZA^{s} =)\,Aq + (Æ \end{cases} \Big\} ZE - E$

Therefore Zq + Eq = ZE + Aq + ZE. *i.e.* 2 ZE + Aq. *Q. E. D.* In numbers thus 36 + 4 = 12 + 16 + 12. Supposing Z to be 6, A 4. E 2.

Where *Note*, that a Square, is a number Multiplied by it self; and a Reactangle, two numbers Multiplied by one another. The like instance in numbers might be given in the other Propositions.

Theorem

Theorem CVII.

The Q of the whole [Z] *and of either of its parts,* [E] *added to it, (as of one line) is* = 4 □s *made of the whole* [Z] *and that same part* [E;] + Q *of the other part* [A].

Viz. Q, Z + E = 4ZE + Aq.
For, Q Z + E[t] $= \underbrace{(Zq + Eq)}_{\overset{u}{=}\ 2ZE + Aq} + 2\,ZE$. *Q.E.D.* [t] *CV.*

[u] *Pre.*

Theorem CVIII.

The Q of the half, [½ Z] *is* = □ *of the unequal parts,* [A + E, *and* B] + Q *of the intermediate part* [E].

Viz. Aq (*i.e.* Q, ½ Z) = A + E × B + E q.
For, A + E × A — E (*i.e.* B) [w] = (A q — Æ + E A — E q *i. e.*) Aq — E q. Therefore, A + E × B + E q = A q. *Q. E. D.* [w] *Sch. CIII.*

Theorem CIX.

The Qs of the unequal parts [Z *and* B] *are equal 2 Qs of the half* [A] *and* *the intermediate part* [E].

Viz. $Zq + Bq = 2Aq + 2Eq.$

x CV. For Zq (*i.e.* $Q\,A+E)^x = Aq + 2\,Æ + Eq$

And, Bq (*i.e.* $Q, A-E)^x = Aq - 2\,Æ + Eq$

Therefore $Zq + Bq = 2Aq + 2Eq.$ *Q.E.D.*

For $+2\,Æ$ and $-2\,Æ$ destroy one another.

Theorem CX.

The Q of the whole [A] *with the piece added,* [E] *and of the piece added, is double to the Q of the half,* $\frac{1}{2}$ A, *and of the half with the piece added* $\frac{1}{2}$ A+E.

y CV. *Viz.* $Zq + Eq = 2Q, \frac{1}{2}A + 2Q, \frac{1}{2}A + E$

z Ax. 5.

a LXXX.

$$\text{For, } \left.\begin{matrix} Zq^y = \\ +Eq \end{matrix}\right\} \left\{\begin{matrix} Aq^z = \left\{\begin{matrix} \frac{1}{2}Aq^a = \!=\!=\!=\!=\!= \\ + \\ \frac{1}{2}Aq^a = 2Q\frac{1}{2}A \end{matrix}\right. \\ + \\ 2\,Æ^z = \!=\!=\!=\!= 4\frac{1}{2}\,Æ \\ + \qquad\qquad + \\ 2Eq \text{———————} \end{matrix}\right\}^y = \left\{\begin{matrix} 2Q\frac{1}{2}A. \\ \\ 2Q\frac{1}{2}A+E \end{matrix}\right.$$

Q. E. D.

Theorem

Theorem CXI.

The Q *of the half with the piece added,* [E,] *is* = *to the* □ *of the whole,* [A] *and the piece added,* + Qs *of the half, and of the piece added.*

Viz. $Q\frac{1}{2}A + E \overset{b}{=} \underbrace{\text{Æ}}_{\overset{c}{\parallel}\ 2\frac{1}{2}\text{Æ}} + Q\frac{1}{2}A + E\,q.$ [b] *CV.* [c] *Ax.* 5.

Q. E. D.

CHAP. IV. PART II.

Of the Power of Proportional Lines.

Theorem CXII.

In Proportional Lines [A . B :: C . D] *the* □ *of the extreams* [A D] *is* = □ *of the middle* [BC.]

DRaw out $\left\{\begin{array}{c}\text{T F}\\ \text{and}\\ \text{H E}\end{array}\right\}$ to G. [d] Because the □ AB is :: CD ([e] since A . B :: C . D) [f] therefore they have the same Diameter L G: [g] therefore

[d] *Def.* [e] *Hyp.* [f] *LXXXIX* [g] *XXX.*

fore the Triangles $\left\{\begin{array}{l} LGH = LGI \\ LKA = LKB \\ KGE = KGF \end{array}\right\}$ there-

fore (the = Triangles being omitted) the □ KH = KI. and the common DC being added

h *Ax.* 8. to these, DC + KH[h] = DC + KI, that is AD = BC. *Q. E. D.*

Scholium.

In $\div\!\!\div$ A, B, C.	The □ of the extreams
4, 6, 9.	= to the □ of the midd-
viz. AC = Bq.	*Note,* This Propositio[n]
36 = 36.	is call'd the **Catholic Th-**
	orem and is the Foundatio[n]

of the *Golden Rule* in Arithmetick.

Theorem CXIII.

In a Rectangled Triangle [h e i.] *the Q of the subtendent* [h, i.] *is = to the 2 Q's. of the 2 sides, viz.* A=B+C

i *LXXVI.* From the ∟ E, let fall the Pp. ED. Th[en] [i] HI, IE, ID $\div\!\!\div$ also IH, HE, HD $\div\!\!\div$

k *LXXXIII.* [k] Therefore, A. $\left\{\begin{array}{c} B \\ C \end{array}\right.$ :: HI. $\left\{\begin{array}{c} ID \\ HD \end{array}\right\}$ But, [l] HI=

l *Ax.* 5.

m *Ax.* 10. $\left\{\begin{array}{c} ID \\ + \\ HD \end{array}\right\}$ [m] Therefore, A = $\left\{\begin{array}{c} B \\ + \\ C \end{array}\right.$ *Q. E. D.*

Theorem CXIV.

In Acute-angled Triangles, the Q. *of one side* [C,] *is less than the* Q's *of the* 2 *other* [Z, B] (*by* 2 ZE.)

viz. Bq + Zq = Cq + 2 ZE. Let fall the (Pp. DE. — n *Pre.*

For, n ||

Dq + Eq + Zq

o || — o *CVI.*

Dq + Aq + 2 ZE

n ||

Cq + 2 ZE. *Q. E. D.*

Theorem CXV.

In Obtuse-angled Triangles, the Q. *of the subtendent of the Obtuse-angle, is* $\sqsubset$ *than the* Qs *of the* 2 *other sides* (*by* 2 Æ.)

Draw out A, to E, and let fall the Pp. DE. That is, Bq = Aq + Cq + 2 Æ.

For, p 11 — p *CXIII.*

Zq + Dq

q 11 — q *CV.*

Aq + 2 Æ + Eq + Dq

p ||

Aq + 2 Æ + Cq. *Q. E. D.*

Theorem

Theorem CXVI.

In Triangles, that Angle is right, the Q. of the whose subtendent is = to the Q's of the other sides.

FOr if the Angle H E I, were Acute or Obtuse, the Q. of the subtendent, H I, would be [r] ⌐ or [s] ∟, then the Qs. of the other sides, contrary to the supposition: Therefore it is ∟. *Q. E. D.*

[r] *CXIV.*
[s] *CXV.*

Theorem CXVII.

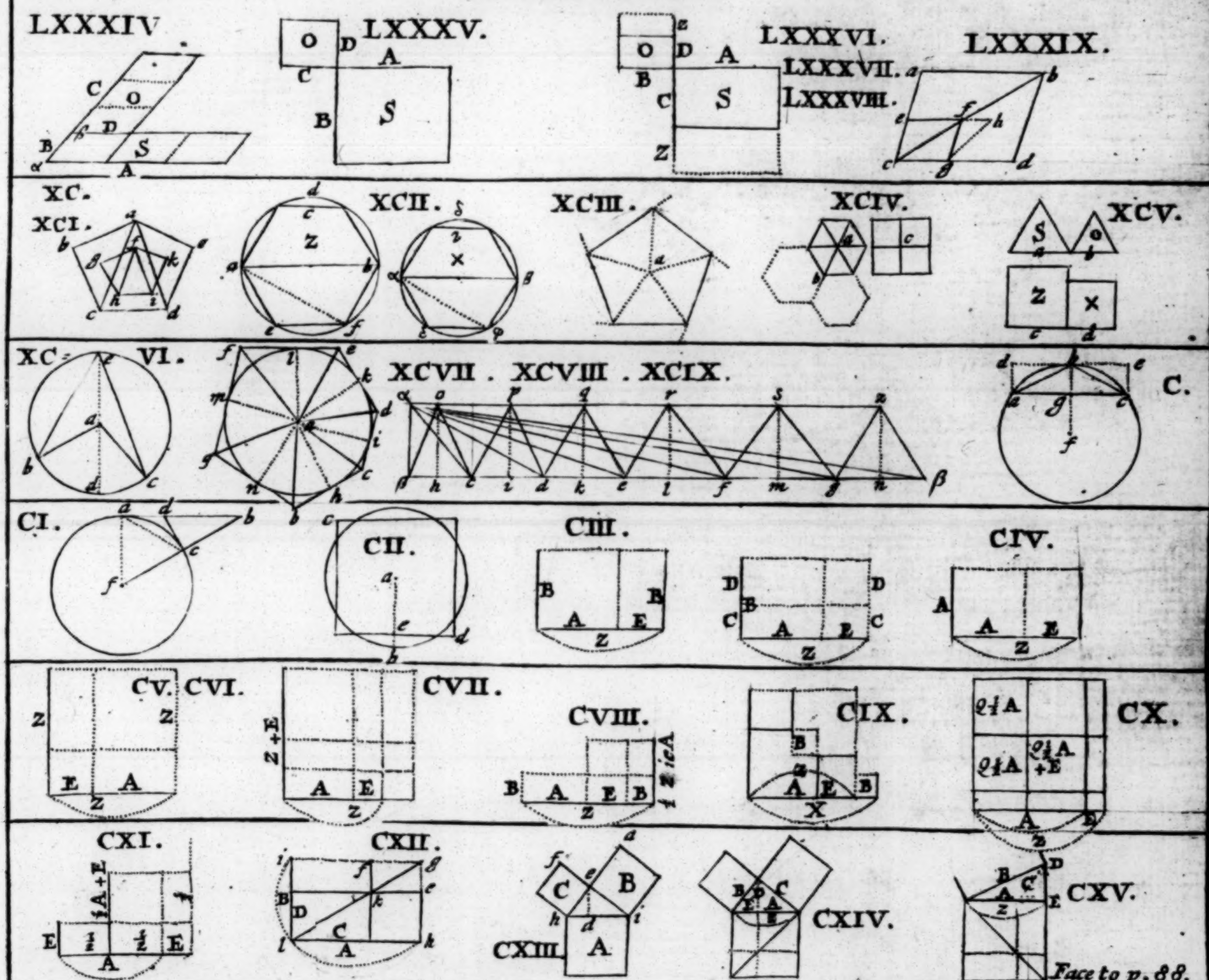
All like Figures made upon the subtendent of an ∟, *are = to those made upon the other 2 sides*, viz. A=B+C.

FOr, A, B, C, are to one another as the Qs. [t] of their Homologous sides, D E, D F, F E. But the Q. of D E is = [u] Q. D F+Q. F E. [w] Therefore the Figure A=B+C. *Q. E. D.*

[t] *XCI.*
[u] *CV.*
[w] *Ax.* 10.

Theorem

THEOREMS.



Face to p. 88.

Of th

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1. BE
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Theorem CXVIII.

The same thing is true of Semicircles.

FOr ○s are to one another as the [x] Q's of their Diameters, [y] therefore also Semicircles. But the Q. of the Diam. F H is = to both the Qs. of the Diam's. F G, G H, therefore the Semicircle B A = BD + CE. *Q. E. D.* *[x] XCII.* *[y] LXIII.*

Thorem CXIX.

The Squaring of the Lunes (or, half moons) of Hippocrates of Scio.

1. BEcause the Semicircle AB [z] = BD + CE; therefore, leaving out the common B and C, there remains the △, A = to the 2 Lunes, D + E. *F. CXVIII.* *[z] Pre.*

2. When the △ is = Legg'd, the Lunes are equal, (the Diameters of the Semicircles being equal) and both together (D + E) are = [a] to the whole △, I K: Therefore either of them are = to ½ the △, *viz.* I or K. *Fig: CIX.* *[a] CXIX. 1.*

3. But when the △ is scalene, it is as difficult to divide it, by the Line G Z, into 2 parts = to the respective Lunes D, E; as to find the Square of a Circle. Of which we spoke before in the Note on Theorem XCIX. *F. CXVIII.*

Theorem

Theorem CXX.

1. *Lines in a Circle*, [a b, d c,] *cut themselves proportionally.* 2. *And their parts make* = ▭s, *viz.* ▭ BEA = CED.

b *LIV.*
c *III.*
d *LXXI.*
e *Def.* like.
f *CXII.*

1. FOr the Angle $\left\{\begin{array}{l} A = D \\ C^{b} = B \\ DEB^{c} = AEC. \end{array}\right\}$ [d] Therefore the △ DBE is like AEC; and by [e] consequence BE . CE :: ED . EA. 2. Therefore the ▭ BEA[f] = CED. *Q. E. D.*

Theorem CXXI.

Lines between Paralls. cut themselves Proportionally, *viz.* A . B :: C . D.

g *LXXI.*
h *IV.*
i *III.*
k *Ax.* 13.

FOr the △ X, is [g] like Z, (because the Angle I = [h] H, and G [h] = K, and the two head Angles are [i] =) and by consequence:

$\left.\begin{array}{l} E.A :: F.B \\ E.C :: F.D \end{array}\right\}$ therefore alterned, E,F :: $\left\{\begin{array}{l} A.B \\ C.D \end{array}\right.$

[k] Therefore A . B :: C . D. *Q. E. D.*

Theorem

Theorem CXXII.

The □ of the whole Secant with the part added, is = to the Q. of the Tangent, drawn from the end of the Secant.

Viz. □ CAE = Q, AD. For the Angle
{ADE[l] = ACD; A, is common:} [m] Therefore AED = [l] *LX.* [m] *XXII.*
CDE: [n] Therefore the △ CAD is like AED; [n] *LXXI.*
Therefore, AC . AD . AE ∺ and by [o] conse- [o] *CXII.*
quence the □ CAE = Q, AD. *Q.E.D.*

Theorem CXXIII.

Lines drawn from a point without, to the inward Circumf. of a ⊙, are to one another, as their parts without the ⊙. viz. A+B . C+D :: D . B.

For the △ HEK is [p] like IGK; ([q] because [p] *LXXI.*
the Angle K is common, and H is [r] = G;) [q] *XXII.*
Therefore A + B . D :: C+D . B; and *Alterned,* [r] *LXX.*
A+B . C + D :: D . B. *Q.E.D.*

Theorem

Theorem CXXIV.

If the Diameter of a ○ [a c] *be cut infinite Perpendiculars, either in, at, without the* ○, *and a Secant* [a e] *drawn, it will be*, AD . AC :: AB . A

[s] *LXXI.* [t] *XXII.* [u] *Hyp.* [w] *LVI.* BEcause the △s EAC and DAB are [s] li (for the Angle ECA [u] = ∟ [w] = AD and A is common,) therefore AE . AC :: A AD, and *Alterned Inverted*, AD . AC :: AB . A *Q. E. D.*

Scholium.

IN the second case (*viz.* at the ○) AC, be the same with AB, AD . AB . AE are ÷ and by consequence the Diameter [AB] i middle Proportional between the whole se [AE,] and the internal part of it [AD].

Theorem CXXV.

A Chord [a d] *dividing equally an An* [b a c] *in a Segment is* (*it self*, *its lesser part*) *reciprocally proportiona the sides of the Angle*, *viz.* AC . AE AD . AB.

[x] *LXXI.* [y] *XXII.* [z] *Hyp.* [a] *LVI.* [b] *Def.* 54. BEcause the △s, ABD and AEC, are [x] li (for [y] the Angles at A are [z] =, and D [a] = [b] therefore AC . AE :: AD . AB. *Q. E. D*

Theor

Theorem CXXVI.

The Q. Circumscrib'd, (or the Q. of the Diameter) is double to the Q. inscrib'd, in a ○, viz. $EFq = 2ECq$.

FOr, EFq (*i.e.* AB) $=$ [c] $CFq + CEq$ (the Angle ECF, [d] being ∟) and EC [e] $=$ CF: Therefore EFq is double to either of them, CFq, or CEq, *i.e.* CD. *Q.E.D.*

[c] *CXIII.*
[d] *LVI.*
[e] *Def. of Q.*

Theorem CXXVII.

A Trapezium (that is, an irregular *four sided figure) being inscrib'd in a Circle: The* ▭ *of the diagonals* [ACDB] *is* $=$ *to the 2* ▭*'s of the opposite sides;* viz. $DC \times AB$, $DA \times CB$.

TAke the $\angle ADE = BDC$. Therefore the △s DCE and DAB, also DAE and DBC are [f] ∷ (For $\angle \begin{cases} DCA \; [g] = DBA. \\ ADB \; [h] = EDC. \end{cases}$ $\angle EDB$ being common, and ADE being [i] $=$ BDC: Also $\angle \begin{cases} DAE \text{ (that is } DAC) \; [i] = DBC. \\ ADE \; [h] = BDC.) \end{cases}$

[f] *LXXI. & XXII.*
[g] *Ax. 8.*
[h] *Const.*
[i] *LIV.*

[k] *Def.* 45. And by [k] consequence $\left\{\begin{array}{l} DC . CE :: DB . BA \\ DA . AE :: DB . BC \end{array}\right.$

[l] *CXII.* Therefore [l] $\square \left\{\begin{array}{l} DC,\ BA = CE,\ DB \\ BA,\ BC = AE,\ DB \end{array}\right.$

[m] *CIII.* Therefore $\square$ DC, BA + DA, BC = (DB, CE + DB, AE [m] =) DB, AC. *Q. E. D.*

CHAP. V.

Of Surfaces.

Theorem CXXVIII.

No part of a Right-Line [bc] *can be out of a Plane, when any part of it, is in it. That is, no Right-Line can lye upon two Planes.*

FOr if you say, that ABC, is part in the Plane, and part out, then by drawing the

[n] *XI.* Line AC, AC will be [n] shorter than ABC, Contrary to *Def.* 4.

Theorem

Theorem CXXIX.

y Right-lin'd △ is in the same Plane. also 2 Right Lines crossing one er.

△ is a Plane Surface, comprehended in three Lines. [o] *Def.* 13.

ierefore, AB, CB are in the same E, CD, [p] will be also in the same [p] *Pre.*

Theorem CXXX.

Right-Lines [a.b, cd] *are in the same Plane.*

ie Right-Lines AD, CB. The [q] △ D is in one Plane; and also the △ it DE and EA are in the same Plane; [q] *Pre.* 1. both the △s are in the same Plane; nsequence their bases, AB, CD. [r] *Pre.* 2.

[k] *Def.* 45. And by [k] consequence $\begin{cases} DC . CE :: \\ DA . AE :: \end{cases}$

[l] *CXII.* Therefore [l] $\square \begin{cases} DC, BA = C \\ BA, BC = A \end{cases}$

[m] *CIII.* Therefore $\square$ DC, BA + DA, BC
CE + DB, AE [m] $=$) DB, AC.

CHAP. V.

Of Surfaces.

Theorem CXXVIII.

No part of a Right-Line [bc] *c*
of a Plane, when any part of i
That is, no Right-Line can
two Planes.

FOr if you say, that ABC, is p
Plane, and part out, then by d
[n] *XI.* Line AC, AC will be [n] shorter th
Contrary to *Def.* 4.

Theorem CXXIX.

1. *Every Right-lin'd △ is in the same Plane.*
2. *And also 2 Right Lines crossing one another.*

1. FOr a [o] △ is a Plane Surface, comprehended within three Lines. [o] *Def.* 13.

2. If therefore, AB, CB are in the same Plane, AE, CD, [p] will be also in the same Plane. [p] *Pre.*

Theorem CXXX.

Parallel Right-Lines [ab, cd] *are in the same Plane.*

DRaw the Right-Lines AD, CB. The [q] △ CED is in one Plane; and also the △ AEB: But DE and EA are in the same Plane: Therefore both the △s are in the same Plane; and by consequence their bases, AB, CD. *Q. E. D.* [q] *Pre.* 1. [r] *Pre.* 2.

Theorem CXXXI.

The interſection [a b] *of two Planes, a Right-Line.*

IF AB, be not right, draw the right AG in the Plane CD; and AHB in the Pla EF; that is, two Right-Lines between the ſa points. Contrary to *Def.* 4.

Theorem CXXXII.

The interſections [a b, CD] *of two Para Planes, by a third Plane, are Para Lines.*

IF not, let AB, CD, meet in I: Therefo the Planes EF, GH, ſhall meet there al [f] (becauſe ABI, and CDI are in the ſa Planes) Contrary to *Def.* 65.

[f] *CXXVIII*

Theore

Theorem CXXXIII.

A Line [dc] *that is Pp. to two Lines* [t] *crossing,* [ac, bc] *is also Pp.* [u] *to their Plane* [af.]

[t] *CXXIX.2*
[u] *Def.* 50.

IF not, let DE be Pp. to the Plane AF, and from the point E, draw the Right-Lines EA, EC, to which, the said DE ought to be Pp. by *Def.* 60.

$$\text{Now, Q, } AD^{a} = \begin{cases} AC^{b} = \begin{cases} AE \\ + \\ EC \end{cases} \\ + \\ DC^{b} = \begin{cases} ED \\ + \\ EC \end{cases} \end{cases}$$

[[a] CXIII. because the Angle ACD, is ∟ by Hyp. [b] because the Angles AEC, dec, ought to be ∟ by *Def.* 60.]

Therefore the Q, AD ⊏ Qs. AE + ED: Therefore the Angle DEA, cannot be ∟. [CXIII.] (the same reason will hold, if the Line DE, touches the Plane any where but in C) and by consequence not DE, but DC [w] is Pp. to the Plane. *Q. E. D.*

[w] *CXVI.*

Theorem CXXXIV.

Three Lines which receive the same P[p.]
[E A] *are in the same Plane.*

[x] *Hyp.* [y] *Def.* 60. [z] *Ax.* 2.

IF not, let A B, be in another Plane [*viz.* E F] cutting the Plane G H, in A F; Since therefore, E A is Pp. [x] to A C, A D, it will a[lso] be Pp. to their Plane, G H: Therefore t[he] Angle E A F, [y] is ∟, as is also [x] E A B, a[nd] by consequence E A F is [z] = E A B; that i[s] the part = to the whole. *Q. E. A.*

Theorem CXXXV.

If of two Parallels, one [a b] *be Pp. to [a] Plane, the other* [c d] *is also.*

[a] *CXXIX.* [b] *Def.* 60. [c] *Ax.*2.*and Hyp.* [d] *IV.*

IF not, let E D be Pp. This will be in t[he] Plane [a] D F. Now, the Angle E D B [b] (∟ [c] = A B I) = [d] C D B, *viz.* part = [the] whole. *Q. E. A.*

Theore[m]

Theorem CXXXVI.

One only Pp. [cd] *can be rais'd from one point in a Plane.*

IF it is possible, let CE be also Pp. and through these two CE, CD, let a Plane be [e] drawn; cutting the other Plane in ACB, the Angles ACE, ACD, must be both [f] ∟; and by consequence =, *viz.* part = whole. *Q. E. A.*

[e] *CXXIX.*
[f] *Def.* 60.

Theorem CXXXVII.

A Pp. [fe] *to one Plane*, [ab] *is Pp. to its Parallel Plane.*

DRaw a Plane through FLE, cutting the others in LH and EG, these intersections will be [g] Parallel. Now the Angle GEF, is ∟ [h], and = [i] HLF: (And the like, if the Plane pass through EM, LO,) [k] therefore EF is Pp. to the Plane, CD. *Q. E. D.*

[g] *CXXXII.*
[h] *Hyp.* [i] *IV.*
[k] *Def.* 60.

Theorem CXXXVIII.

Planes are Parallel that receive the same Pp.

[l] XVI. FOr since the Angle GEF = HLF, [l] there-
fore the Line HL is Parallel EG, (and th
like of ME to OL, &c.) wheresoever the i
tersection can happen; therefore the Pla
AB is Parallel to CD. *Q.E.D.*

Theorem CXXXIX.

*If two crossing Lines, [bac] are Para
to two crossing Lines [edf] in anoth
Plane, their Planes also will be Pa-
rallel.*

[m] Hyp. DRaw AG Pp. to the Plane EF, and HG
[n] Def. 60. Parallel (EDF, [m] Parallel) BAC: There-
[o] VI. fore the Angles HGA, IGA [n] are ⌊ [o] =
[p] Constr. CAG, BAG: Therefore AG is Pp. to t
[q] CXXXIII Plane [p] EF and [q] BC: And by [r] consequen
[r] Pre. the Planes BC, EF are Parallels.

Theore

Theorem CXL.

Two meeting Lines [bac] Parallel to two [edf] in another Plane, contain an = Ang. with them, viz. *Ang.* A=D.

TAke AB = DE, and AC = DF, and joyn BE, AD, CF: Since AB, ED, and AC, DF, are [f] Parallel and [t] =; BE is alſo [u] = and Parallel, (to AD [u] = and Parallel) [w] CF. Therefore BC [u] = EF; and by conſequence the △BAC [x] = EDF, and the Angle A = D. *Q.E.D.*

[f] *Hyp.*
[t] *Conſtr.*
[u] *XXX.*
[w] *VIII.*
[x] *XVI.*

Theorem CXLI.

Parallel Lines [ae, bf] are cut proportionally by Parallel Planes.

Viz. AC . CE :: BD . DF: Becauſe the interſections AB, CD, EF, are [y] parallels; therefore $\frac{BD}{DF} = \left(\frac{BZ}{ZE} = \right)\frac{AC}{CE}$. *Q. E. D.*

[y] *CXXXII*
[z] *LXVII.*

Part II.

PART. II.

Of a Solid Angle.

Theorem CXLII.

Of three Angles which are necessary to compose a Solid Angle [a] *any two,* ... ⊏ *than the third.*

IF three Angles are =, it is evident: B... one be biggest, viz. BAC, take BAE... BAD; joyn BC, and by the Lines BD, C... take off AD = BE: Because, AD = A...
a consfr. and AB is common, and the Angle BAE...
b XVII. BAD: [b] Therefore BD = BE; but BD...
c XI. DC [c] ⊏ BC: Therefore (BD being [a] =...
d Ax. 9. DC [d] ⊏ EC; and by consequence the An...
e XIX. DAC [e] ⊏ EAC; therefore (BAD being... BAE) BAD + BAC [d] ⊏ BAE + EA... i. e. BAC. Q. E. D

Theorem CXLIII.

A Solid Angle [a] *is made up of less than Four* ∟ *Angles.*

FOr, the 6 Angles at B, C, and D, +
f X. the 3 at A, are [f] = 6∟; but the 6, a...

and D, are [g] ⊏ than (the 3 B, C, D, in the ſe of the Pyramid [f] =) 2 ∟: Therefore the at A, are ⊐ 4 ∟. *Q. E. D.* [g] *Pre.*

The reaſon is this, ſince the Four Angles ade by 2 Lines croſſing, (as at Z) are [h] = ∟; The Plane [i] in which they are cannot be olded (in the Lines Z F, Z G, Z E, Z H) as make a Solid Angle at Z, unleſs ſomthing left out from one of the Angles, as F Z S. [h] *I.* [i] *CXXIX.*

CHAP. VI.

Of Bodies, or Solids.

Theorem CXLIV.

The oppoſite Planes [a c, d b] *of a Parallelepipedon (Ppp.) are like and equal.*

AC, DB, are [k] Parallel: Therefore the interſections A F, D E are [l] Parallel; and FDE is a [m] Pgrm. by conſequence A F [n] = E. After the ſame manner it may be prov'd at all the other oppoſite Lines are Parallel nd =: Therefore the Angle F A H = E D G, nd A F C = D E B. Laſtly, the Plane A C [o] = and [p] like D B, and ſo of the reſt. *Q. E. D.*

[k] *Def.* [l] *CXXXII.* [m] *Def.* 20. [n] *XXX.* [o] *XXXIV.* [p] *Def.*

Theorem

Theorem CXLV.

A Ppp. is divided in the middle, by** **Plane** [a h] **that passes through the D** **ameters** [ea, hm.] **of two opposite Pla

[q] *Pre.*
[r] *XXX.*

ALL the opposite Pgrs. are [q] =: The AED [r] = AEF, and LMH [r] = MH the Plane AH is common, therefore the P or Prism AL = AG. *Q. E. D.*

Theorem CXLVI.

Ppps.** [ZX **and** ZS] **are = which b the same base and height.

(*Note*, This Figure would be best comprehe ded, if it were cut out in Cork, or the matter.)

1. LEt them fall between the same Paralle KF, and DA: Therefore the Pr (EDH, *i.e.*) S + O = O + X. [And (the co mon O, being left out) S = X, and (Z be added) S + Z = X + Z. *Q. E. D.*]

[s] *XXIV.*
[t] *XXX.*
[u] *IV.*
[w] *CXLIV.*
[x] *Hyp.*
[y] *XXXIV.*
[z] *Ax.* 8.
[a] *Def.*

Because, the Plane 1. EPH [s] = FRA, EH ing [t] = FA, EP [t] = FR Ang. P = [u] R, 2. Planes opposite to these, are = by the sa reason. 3. DE [w] = GF, and LE [y] = CF, ing between the same [x] Parallels, and the ba KE [t] = MF. 4. GA = DH; because DR (KF [w] =) LA: Therefore DR + GH [z] = L + GH. [a] Therefore S + O = O + X. 2

2. If a Ppp. of the ſame baſe and height with Z X, ſhould not fall between the ſame Parallels; It will however fall between the ſame Parallels with S Z; and being, as before, = prov'd to it, will be [b] = to Z X. [b] *Ax.* 6.

Theorem CXLVII.

Ppps. [S Z, O] *are = which have an = and like baſe, and the ſame height.*

UPon the baſe C A B, ſuppoſe a Ppp. fram'd Z X, of the ſame height, and like to O. Then, O = Z X, (for the ſeveral ſides will be prov'd = by XXXIV. and *Like*, by conſtruction) which Z X [c] = S Z; therefore O [d] = S Z. *Q. E. D.* [c] *Pre.* [d] *Ax.* 6.

Theorem CXLVIII.

Ppps. [a b, a d.] *of the ſame height, are to one another as their baſes.*

LEt the baſe C G, be made = C E; therefore the Ppp. S = A H: Now Let the Plane B F, (being [e] Parallel G H and A C) be mov'd Parallely from A C to G H. It ſhall at the ſame time diſpatch the baſe G C, and the Ppp. A H: And by conſequence how much ſoever it has taken away at any time from the baſe, it will have taken away a *Like* part from the Ppp. So that, as the baſe G C is (for inſtance) to its part G F : : Ppp. A H (= S X.) *Q. E. D.* [e] *Def. Ppp.*

Theorem

Theorem CXLIX.

Def. 46.

Equal Ppp's. [S = XO] *have their bas… and height, reciprocally proportiona… viz.* AB . CD :: MK . AI.

f Pre. g Ax. 10. because O + X = S. Hypo. h LXXX.

TAke the Ppp. X, of the same height as S… that is, ML = AI. Then, $\frac{AB}{CD}$ f = (…

g = $\frac{O+X}{X}$ f = $\frac{DK}{DL}$ h = $\frac{MK}{ML}$; that g is) $\frac{M…}{A…}$ (because ML = AI, Constr.) *Q. E. D.*

Theorem CL.

a Def. 46.

Those Ppps. [S = XO] *are equal, wh… bases and heights are* a *reciprocally pr… portional, viz.* AE . FC :: DN . BE…

i CXLVIII. k Sup. l LXVII. with LXI. 3 m LXXX. n Ax. 10.

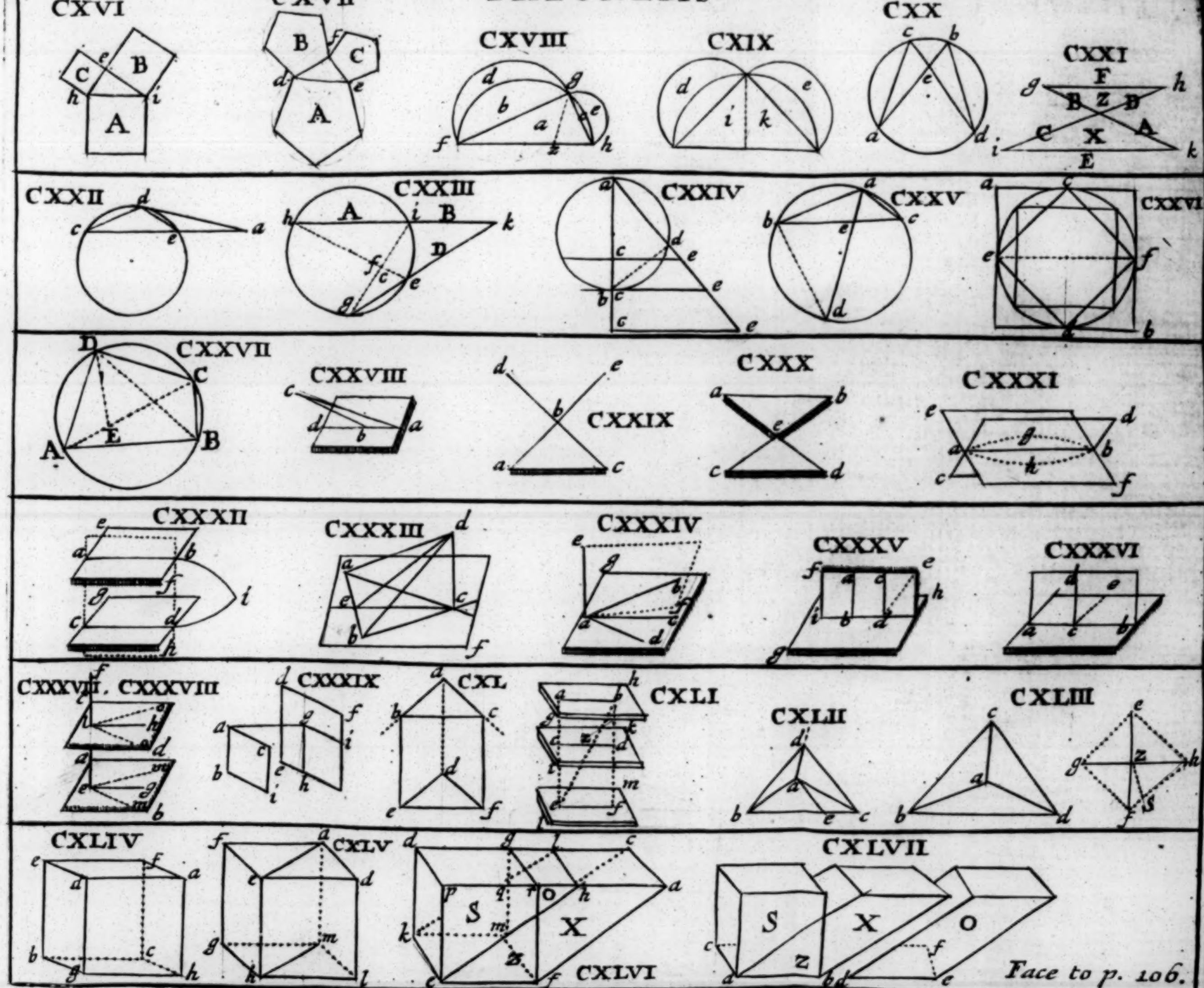
FOr, Ppp. $\frac{S}{X}$ i = (*bas.* $\frac{AE}{FC}$ k = *height* $\frac{DN}{BE, i.e. N…}$

l = *side* $\frac{DF}{HF}$ m = *Pgr.* $\frac{DK}{HK}$ i =) $\frac{XO}{X}$. Therefo…

S n = XO. *Q. E. D.*

Theor…

THEOREMS



Face to p. 106.

Def. 4(

f *Pre.*
g *Ax.* 1C
because (
$+ x =$
Hypo.
h *LXX*

a *Def.* 4

i *CXLVI*
k *Sup.*
l *LXVI.*
with LX
m *LXX*
n *Ax.* 1C

Like Ppp
reason
S . O

BEcause t
PAE,
up the

$\frac{S}{X} = \Big(\frac{A}{E}$

$=$ —

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gain, $\frac{L}{H}$

$= \frac{HE}{EF}\Big)$ S

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$\Big(\frac{LE}{IN} =$

; that is

$= \Big($

Q. E. D.

Theorem CLI.

Like Ppps. [S :: O] *are in a triplicate reason of their homologous sides, viz.* S . O :: AE . ED. *thrice.*

Because the $\angle$ DEF° = AEH, therefore ᵖAE, ED may be plac'd in a Right Line, ° *Def.* 45. ᵖ *III.*
...up the void space with X and Z: Now,

$$\frac{S}{X} \overset{q}{=} \left(\frac{AH}{EI} \overset{r}{=}\right) \frac{AE}{ED} \quad \text{Also, } \frac{AE}{ED} = \frac{LE}{IN} = \frac{HE}{EF}.$$

$$\frac{X}{Z} = \text{———} \overset{r}{} \frac{LE}{IN} \quad \left(\text{For } \frac{AE}{LE} \overset{s}{=} \frac{ED}{(FC, i.e.)IN}\right.$$

$$\frac{Z}{O} = \text{———} \overset{r}{} \frac{HE}{EF} \quad \text{and } \textit{Alterned } \frac{AE}{ED} = \frac{LE}{IN}.$$

ᑫ *CXLVIII* ʳ *LXXX.* ˢ *Def.* 45.

Again, $\frac{LE}{HE} = \frac{(FC\ i.e.)\ IN}{EF}$, and *Alterned* $\frac{LE}{IN} = \frac{HE}{EF}$) So then it appears that, $\frac{S}{X} = \left(\frac{AE}{ED} = \frac{LE}{IN} =\right) \frac{X}{Z} = \left(\frac{LE}{IN} = \frac{HE}{EF} =\right) \frac{Z}{O}$: Wherefore $\frac{S}{X} = \frac{X}{Z} \overset{t}{=} \frac{Z}{O}$; that is, S, X, Z, O ÷÷ and by consequence ᵗ *Ax.* 10.

$\frac{S}{O} \overset{u}{=} \left(\frac{S}{X}\right.$ thrice, which $\frac{S}{X} = \left.\right) \frac{AE}{ED}$ thrice. ᵘ *Def.* 42.

Q. E. D.

Scholium,

Scholium to this, and the former Theo.

BEcause Ppps. are extended in length breadth and depth; therefore th the reason of one Ppp. to another, m be known, we are to take the reasons all the dimensions together, viz. *if a Pp* A, *be twice as broad, three times as lon and four times as high as* B; *then* A *twice* 3 *times* (*i. e.* six times) 4 *tim* (*i. e.* 24 times) *as great as* B. *But the Ppps. are* like : *Then the reasons their length, breadth and depth are th same* [a] : *So that if* A *be twice longer it must be also twice broader and twic higher than* B. *That is, twice twi* (*i. e.* 4 times) *twice* (*i. e.* 8 times *greater than* B. *So that the same reas is taken* 3 *times; and is thereupon call* Triplicate Reason.

[a] *Def.* 45.

Theorem

Theorem CLII.

Like Ppps. are to be one another, as the Cubes of their Homologous *sides.*

BEcause all Cubes are *like* Ppps. and by consequence are to one another in a Triplicate reason of their sides (by Prec. Theor.) But like Ppps. are in the same [a] reason. Therefore they are to one another as the Cubes rais'd upon their Homologous sides. *Q. E. D.*

[a] *CLI.*

Theorem CLIII.

WHat has been said in the foregoing Propositions concerning Ppps. does agree also to Prisms. (as being the [a] halves of a Pp) Provided that their opposite Parall. Planes, if they are not $\triangle$s, be resolv'd into $\triangle$s.

[a] *CXLV.*

Theorem CLIV.

They agree also to Cylinders. For Example. (according to CXLVI) *If Cylinders have the same base* [G B] *and equal height then they are equal.* viz. **A B=B C.**

IF not; let A B be ⊏; and let there be inscribed in it a Prism of many sides (as A D F E) ⊏ also then the Cyl. B C. (Because by multiplying the sides of the Prism, you may approach by infinite degrees nearer and nearer to the Cyl. in which it is inscrib'd. Besides that, though the degree by which the Cyl. B C. is less than B A, be supposed to be never so small, yet it is stated and fixt, and may not (after it is given) be alter'd. Whereas our Approaches in the Prism may be made to Infinity.) Then upon the same base, G F, let there be inscrib'd a Prism in the Cyl. B C; These Prisms will be [a] equal, that is, the Prism H C F (=D A F) ⊏ [b] Cyl. B C. A part greater then the whole. *Q.E.A.*

[a] *Pre. & CXLVI.*
[b] *Constr.*

Theorem

Theorem CLV.

A Plane [l m n] *cutting a Pyramid* [b d] *Parall. to the base* [a b c] *makes a figure* like *the whole.*

FOr the Ang's at the top are common, and those at the bases, may be proved = by Theor. IV. Then, that the sides about these respective <s are proportional, appears thus; A B . L N : : [a] A D . L D; and alterned; A B . A D : : L N . L D and so of the others. Wherefore the figures are [b] like. *Q. E. D.*

[a] *LXIX.*

[b] *Def.* 45.

Theorem CLVI.

The Sections [l m n o p r] *of two Pyramids (whose bases, and heights are =) made by a plane* [l n r p] *Parall. to both their bases, are equal.*

BEcause the bases, A B C, E F G are : : [a] the Sections L M N, O P R; therefore $\frac{ABC}{LMN}$ and $\frac{EFG}{OPR}$ in a duplicate [b] reason of their Homologous sides, but A B C is = [c] E F G. Therefore, L M N = [d] O P R. *Q.E.D.*

[a] *Prec.*

[b] *XCI.*

[c] *Sup.*

[d] *Ax.* 10.

Theorem CLVII.

All Pyramids are equal, which have equal bases and height.

FOr instance C D = F H. If not, let C D be $\sqsubset$; and inscribe in this a solid Segment, (made up of Prisms having *like* bases and = height, L N M) $\sqsubset$ than the Pyram F H. (In which we argue, as before we did in Th. CLVII. because, the height of the Prisms being lessen'd, they may be infinitely multiplied, still approaching toward the Pyram. in which they are inscrib'd; till at last they may be made to come nearer to it than any other quantity that has been already given, as F H.) Then let there be inscrib'd also in F H, as many Prisms (for there can be no bound to the number.) Both these Segments of Prisms shall be =; (for their number is =, *by Construction*; Their bases and heights may be equal, because those of the Pyramids in which they are inscrib'd, are = *by Supposition*) that is, the Prz. in H F (= Prz. in D C) $\sqsubset$ [a] Pyram. H. F. *Q. E. D.*

[a] *Constr.*

Theorem

Theorem CLVIII.

Every triangular Prism *(that is, which has a* △ *for its base) may be divided into 3 equal Pyramids*; viz. EZD,A. ABC,D. AFC,D.

FOr the base ABC = [a] EFD; and the height is equal.[a] Therefore, ABC,D = [b] EFD,A = AFC, D. Because the base AEF = [c],[a] AFC. and the height at D is common. *Q. E. D.*

[a] *Def.* 61.
[b] *Prec.*
[c] *XXX.*

Note, This Figure will be plain, if cut out in Cork.

Theorem CLIX.

WHat has been demonstrated of *Prisms*, is true also of *Pyramids*, as being the third part of a *Prism*. [a]

[a] *LXIII, & Prec.*

Theorem CLX.

What has been demonstrated of Pyramids *(in the Precedent Theor.) agrees also to* Cones.

FOr instance. The Cone ABC = DEF; having = bases and height. If not, let

A B C, be ⊏; and let there be inſcribed in this a Pyram. A G H C, ⊏ Cone D E F. (which, that it may be done, we prove by multiplying the ſides, as we have done before,) Then (becauſe the baſes of the Cones are = [a]) let there be inſcrib'd in the Cone D E F, a Pyram. E D K L M = [b] and : : A G H C. Now, the Pyram. E D K L = (Pyr. A G H C) ⊏ [c] cone E D K. that is, a part ⊏ whole. *Q. E. A.*

[a] *Sup.*
[b] *Prec.* being the ſame height.
[c] *Conſtr.*

Theorem CLXI.

All like *Bodies are in a Triplicate Reaſon of their Homologous ſides.*

BEcauſe they may be reſolv'd into Triangular Pyramids, equal in number, [a] and like. [a] But ſuch △r Pyramis: are in a [b] Triplicate Reaſon of their homol. ſides; therefore [c] alſo the Bodies. *Q. E. D.*

[a] *Def.* 45.
[b] *CLIX*
CLIX.
[c] *LXIII.*

Theorem CLXII.

A Sphere *is = a* Cone *whoſe perp. axis is the radius of the* Sphere, *and its baſe = the whole ſurface of the* Sphere.

THe ſame Demonſtration ſerves here as in Theor. XCVII. by ſhewing that all Polygons circum-

circumscrib'd, or inscrib'd in a *Sphere* are $\sqsubset$ or $\sqsupset$ than such a *Cone*. Therefore the *Sphere* is $=$ to such a *Cone*. *Q. E. D.*

Theorem CLXIII.

Of all solid figures (having an $=$ surface) The Sphere *is the greatest.*

THis also is demonstrated from the Precedent, by Theor. CII.

Theorem CLXIV.

Spheres are in a Triplicate Reason of their Diameters; *or, as the Cubes of their Diameter's (Theor.* CLI. CLII.) viz.

$$Sph.\ \frac{ABC}{DEF} = \frac{AC}{DF}\ thrice.$$

IF not, let the Sphere $\frac{ABC}{DEF}$ be $\sqsubset$ $\frac{AC}{DF}$ thrice; and let there be inscrib'd in the Sph. A B C a solid figure of many sides, A B C G, $\sqsubset$ to the side D E F than A C to D F *thrice*. (since by continual doubling [a] the sides of the body inscrib'd, as

[a] *Theor.* C.

as AI, IB, &c. whereby the same line AC will always continue to be both the Diam. of the ○, and the side of half the Polygon, we may approach still nearer to the Sph. ABC, and by consequence, make it bigger than any other body, which is less than the Sphere) Then let there be inscrib'd in the other Sphere a *like* body of many sides. These bodies, being resolv'd into Pyramids and so into △s. [b] will be in a Triplicate Reason of their homol. sides. AC, DF, *viz.* the [c] Body ABCG. DEFH : : AC. DF *thrice*. But $\frac{ABCG}{DEF}$ ⊏ [d] $\frac{AC}{DF}$ *thrice*. Therefore the Sph. DEF, is ⊐ than the Body inscrib'd in it, DEFH. *Q. E. A.*

[b] *CLXI.*
[c] *Ax.* 14.
[d] *Constr.*
[e] *Ax.* 11.

Theorem CLXV.

There can be but 5 regular *Bodies*, (*which have all their sides, and all their Angles* =)

FOr no plane figures, joyn'd together, can make a solid Angle; except a △, □ and *Pentagon*. For a solid Ang. ought to consist of less [a] than 3 ∟s. and 3 plane ∠s are the fewest that can make up a solid one (as is naturally evident.) Now 3 ∠s of a Hexagon are = 4 ∟; and in all figures of more sides than a Hexicon. they exceed 4 ∟. Wherefore 3, 4, and 5 △'s

[a] *XLIII.*

△'s (for 6, make [b] 4 ∟s) 3 □, and 3 *Pentagons* (which make [c] $\frac{9}{10}$ of 4 ∟s) may all compose a solid Angle. Upon this account there can be but 5 *regular* Bodies. *Viz.*

[b] *XCIV*, 3.
[c] *XCIV*.

1. *A Tetraëdrum.* [d]	Whose Angle is made up of	3	Equilateral △s.
2. *An Octaëdrum.*		4	
3. *An Icosaëdrum.*		5	
4. *A Hexaëdrum.*		4	□s
5. *A Dodecaëdrum.*		3	*Pentagons.*

[d] *Def.* 74. &c. *Fig.* CLXIV.

If these, or *like* figures, be cut in Past-board according to the figures in the Type, and folded up, they will represent the foresaid 5 regular Bodies.

Theorem CLXVI.

It will not be amiss in conclusion of the Theorems, to add one Proposition that may serve as an Introduction to the Doctrine of Infinites. *viz. That* Infinites *may be* actually *number'd or measur'd.*

IF from the line A B you take (*suppose*) a fourth part toward A; [A C] and again towards B, 2 such parts, [D B] (*viz.* such a number of parts, less by 2, than the whole line was first supposed divided into) there will remain C D, one fourth of A B. If again, from this *remainder,*

der, you take as before, one part towards A, [C E] and 2 such towards B, [F D] there will remain only E F one fourth of C D. And so if you continue to do to every remainder, there will always remain between the lines last taken, one fourth of the line from whence they were taken. From which fourth part, there may still after the same manner be supposed to be taken two other such lines on each side; but if this be done infinite times *actualy*, then there will nothing more remain (between) and so the continu'd division on either side will come exactly to the point G; which cuts off a third part of the line A B. (*viz.* a part, of one Denomination less, than the whole line is first divided into.) Because there was always taken away twice as much towards B as towards A. The total sum, therefore, of all together that is taken away towards B, will be twice as much as the Total Sum towards A. And by consequence, the *meeting* will be at such a point [G] as cuts off G B double to G A. Which was the thing undertaken; viz. *to assign the precise measure of a line infinitely divided.*

But if it be inquir'd how this division can be made infinite times *actually*; I answer; Let two points begin to move from A and B, at the same time; That at B, always moving as fast again as that at A. It is certain they will at length meet; and (by the former Demonstration) just at the point G. For the point B, will be at D, when A is at C. and B again at F, when A is at E. and so they will each pass through the infinite Sub-divisions before mentioned; and when they meet, will have divided the line A B infinite times *actually*.

The

The same may be said, if one of the points be to overtake the other; as in the famous Sophism of *Zeno*, who argu'd that a Horse would never overtake a Snail. For suppose the Horse run ten times as fast as the Snail, and the Snail were a Mile before, and both set out at the same time. When the Horse has run the first Mile, the Snail will have got to the 10th part of the 2d Mile; when the Horse has run this 10th part, the Snail will be got to the 10th of the next 10th part; (that is, a 100th part of the 2d Mile) and when the Horse has got this, the Snail will be a 1000th part before him. And so in infinite Subdivisions, when the Horse has got the last part, the Snail will still have got a part before him. On the contrary, it is naturally evident, as any Axiom, that the Horse will at length overtake the Snail; and (by consequence) measure out those infinite Sub-divisions *actually*; The supposition of the Impossibility of which, is the ground of the Fallacy. Now the precise point of meeting may be determin'd, by a very easy Demonstration. *viz. at the end of the 9th part of the 2d. Mile.* For since the Horse runs 10 times as fast as the Snail, the Horse will run $\frac{10}{9}$ of a Mile, while the Snail runs $\frac{1}{9}$; and by consequence they will come both at the same time to the end of the first 9th part of the 2d Mile. *Q. E. D.*

Theorem

PROBLEMS

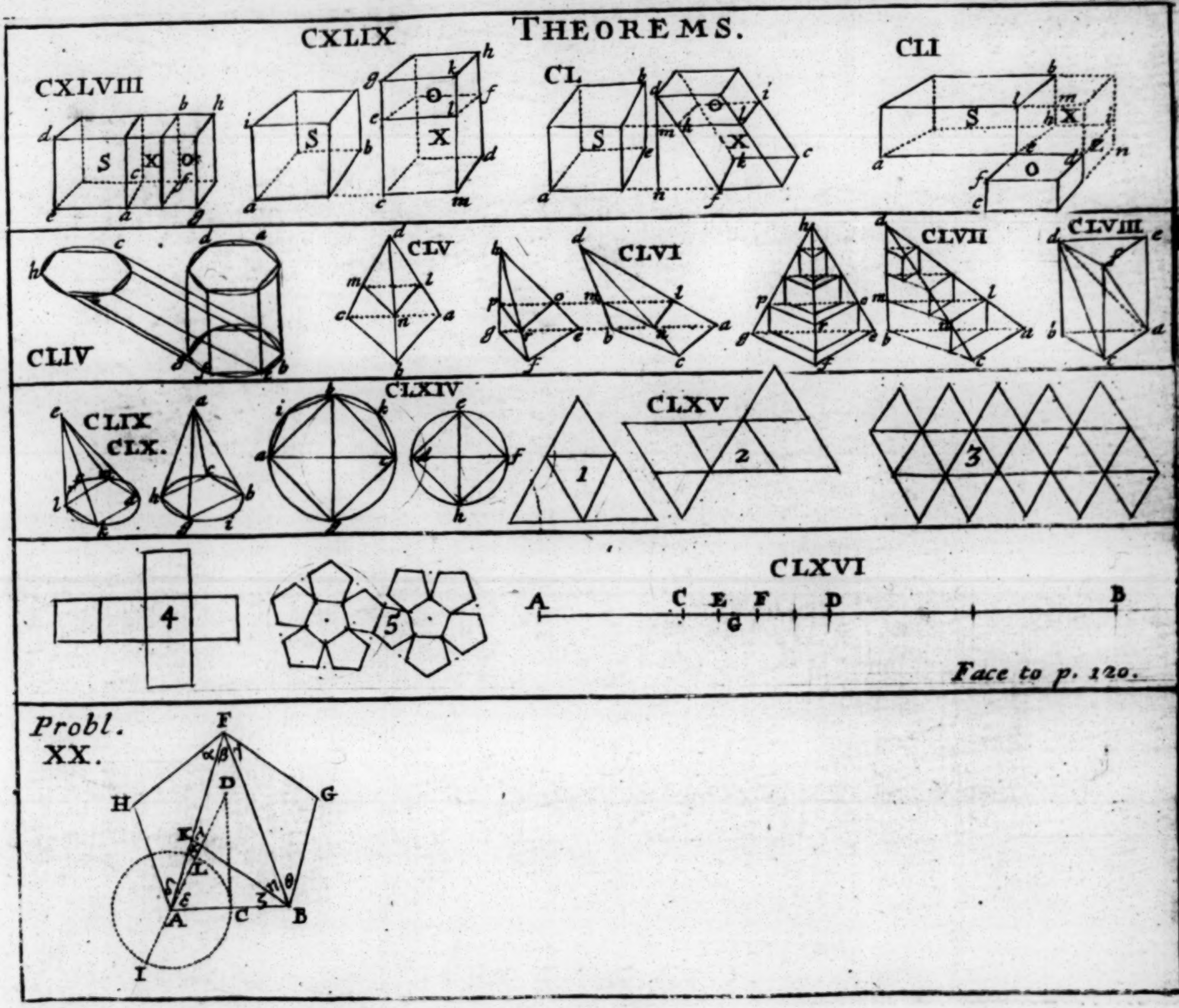
Which purpose something to be done.
Demands, or Suppositions.

1. *That a right line may be drawn from any one point, to another.*
2. *That a right line may be continued at either end as far as we please.*
3. *That a Circle may be describ'd upon any Center, and at any distance, (or interval.)*

Problem I.

From a point given [a] *to draw a right line* [a c] *parall. to another* [b c].

ON the Cent. A, at any distance, (so as to cut the line B C) describe the Arch, D G E. Cent. D, same distance describe another Arch cutting the line given (B C) in G. Lastly, Cent. G same distance, cut the first Arch in E, the line


THEOREMS.
CXLVIII
CXLIX
CL
CLI
CLIV
CLV
CLVI
CLVII
CLVIII
CLIX
CLX.
CLXIV
CLXV
1
2
3
4
5
CLXVI
A
C
E
G
F
D
B
Face to p. 120.
Probl.
XX.
F
H
G
D
K
L
A
C
B
I

AE is parall. BC. For AD = [a] AE = [b] G = [b] GE, and AG is common. There-in the △'s AEG and AGD, the ∠GAE [c] AGD. Therefore [d] AE is parall. to BC. E. F.) *which was to be done.*

[a] *Radius's of the same* ○. [b] *Rad's of* = ○s. [c] XIII. [d] *VI.*

Problem II.

om a point given, [a] *to draw a line,* [a c] *equal to a line given* DG.

om the point A, draw AE parall. [d] to DG, join the points AD. From the point G, draw parall. [d] to AD. The line AE is = DG, ause in the Pgr. [e] DE, the opposite sides [f] E = DG. *Q. E. F.*

[d] *Preced.* [e] *Def.* 20. *by Constr.* [f] *XXX.*

Problem III.

om a line given, [a b] *to cut off a part* [a f] *equal to another given* [c d].

Rom the point A, draw AE = [g] CD, Cent. A. interval CD, (or AE) describe the Arch F. The line FA is = CD, because FA = [h] E = [i]) CD. *Q. E. D.*

[g] *Prec.* [h] *Radius's.* [i] *Constr.*

Problem

Problem IV.

To make an Ang. [g] = *to an An*
given [b].

k *Prec.* FRom the line L F, take the part k L D
A B and D E = A C and E F = C B, th
on the Cent. $\left\{\begin{matrix} D \\ E \end{matrix}\right.$ interv. $\left.\begin{matrix} D\,L \\ E\,F \end{matrix}\right\}$ describe Arc
l *XVI.* cutting in G, joyn G D, G E. The Ang. G
m *Constr.* = B, (because the △ D G E is equal m sided
& *Ax.* 6. A B C.) *Q. E. F.*

Problem V.

To divide an Ang. given, [b] *in two equ*
parts.

CEnt. B, at any Interv. describe the Arch A
Cent's. A and C, same Interv. desc. Arc
n *XVI.* crossing in D, joyn D A, D C, and draw D
which divides the Ang. D in two = n par
(For the △s B A D, B C D are equal sided,
sides B A D, B C D, being Radius's of = (
and D B common, therefore the Ang. C D B
= A D B.) *Q. E. F.*

Probl

Problem VI.

From a point given, [a *or* d] *to raise, or let fall a Perp.* [a d].

I. FRom each side of D take equals,[o] D B, D C. Cent. and interv. B, C, describe Arches crossing at A, joyn A D, which is the Perp. required. For A B is = [p] A C, D B = [q] D C, D A common. Therefore the Angles at D are = [r], by conseq. ∟ [s] and D A is perp.[t]

II. Cent. A, at any interv. descr. the Arch B C, joyn A B, A C, divide the ∠ A in the [u] middle, by the time A D which is the perp.[t] required. For the Angles at D, may be proved =, as before. Therefore, &c. *Q. E. F.*

[o] *Prob. III.*
[p] *Radius's.*
[q] *Constr.*
[r] *XVI.*
[s] *I.*
[t] *Def. 7.*
[u] *Pr. V.*

Problem VII.

To divide a line [a b] *in the middle.*

CEnt. A B, any interv. describe Arches crossing in D, let fall the perp. D C [w]. This divides A B in the middle. For A D = [x] B D . D C, com. the Angles at D are = [y]. Therefore the side C A is = [z] C B. *Q. E. F.*

[w] *Prec.*
[x] *Radius's.*
[y] *Def. 7.*
[z] *XXIX.*

Problem

Problem VIII.

To divide a line [a b] *in a given Proportion.* $\frac{CA}{EA}$

AB, AC, being placed at any Ang. A, joyn BC, and from the point E, draw a Parall. [a] to it, *viz.* ED. Therefore [b] BA . DA : : CA . EA. *Q. E. F.*

[a] *Pr. I.*
[b] *LXVII.*

Problem IX.

To find a fourth proportional, [d e] *to three given,* viz. AB . BC : : AD . d e

JOyn BD, and from the point C, draw its Parall. [c] CE, joyn DE, the fourth proport. for [d] AD . BC : : AD . DE. *Q. E. F.*

[c] *Pr. I.*
[d] *LXVII. divided.*

Problem X.

To find a third Proport. [d e] *to two given,* [a b . b c]

TAke AD = [e] BC, and joyn it to the point A, at any Ang. draw DB, and its Parall. [f] CE, from the point C. DE is the third Proport. for [g] AB . BC : : (BC, *i. e.*) AD . DE. *Q. E. F.*

[e] *Pr. II.*
[f] *Pr. I.*
[g] *LXVII divided.*

Problem

Problem XI.

To find a middle proportion [b c] between two given [a b b c.]

FRom the point B raise the perp. [h] BE. divide AC in the [i] middle; D. Cent. D, interv. describe a Semicirc. cutting the perp. in E. EB will be the middle proport. For (EA, EC, being joyn'd) the ang. AEC is [k] ∟ therefore the perp. EB is a mid. propor. between the parts of the base ABC. *Q. E. F.*

h *Pr. VI.*
i *Pr. VII.*
k *LXI.*
l *LXXVI.*

Problem XII.

To divide a line given [Z] in extreme and middle reason. So that Z, A, E ∺ and Z × E = Aq.

FRom C, raise the perp. [m] CB = [n] Z. divide CB in the mid. in D. take [o] DF = [n] DH; and CG = CF. G, is the point of division.

For, ▭ BCF + CFq + DCq [p] = (DFq / DFq *i.e.* [q] DHq [r] =) Zq + DCq therefore (omitting the com. DCq) remains BCF + CFq *i.e.* [q] ZA + Aq = Zq

But Zq [f] = ZA + ZE; therefore (ZA being com.) ZE is = [t] Aq, and by consequence [u] Z, A, E, are ∺. *Q. E. F.*

m *P. VI.*
n *P. III.*
o *Dem. II.*
p *CXI.*
q *Constr.*
r *CXIII.*
f *CIV.*
t *Ax.* 8.
u *LXXXI.*

Problem XIII.

1. *To make an equilateral triang.* [a b c.]
2. *To make a triang. of lines given.* [d a b e.]

1. DRaw AB. Cent. A and B describe arches crossing in C. joyn CA, CB. therefore [w] all the sides are = *Q. E. F.*

[w] *Radius's of = ○s.*

2. Cent. A and B; interv. AD, BC, describe arches crossing in C. joyn CA, CB. therefore [w] AC = AD, and CB = BE. *Q. E. F.*

Problem XIV.

To make a Pgr. ([g h] *at an ang. given* [d]) = *to a* △ *given* [a b c.]

[x] *P. I.*
[y] *P. VII.*
[z] *P. IV.*
[a] *XXX.*
[b] *LXXVIII*
[c] *LXIII.*

DRaw BH. [x] pall. AC. divide AC in the mid. [y] G. and make the ang. CGF = [z] D. draw CH pall. GF. and the diag. GH. (The △ GHC [a] =) $\frac{1}{2}$ pgr. FC is [b] = $\frac{1}{2}$ the △ ABC. therefore the whole pgr. FC is [c] = whole △ AB.C. *Q. E. F.*

Problem

Problem XV.

To make a Pgr. [x] (*on a side,* [a b] *and ang.* [c] *given*) = *to a* △ *given* [D.]

MAke S =[d] D, the ang.[b] being *d* = *c.* on the line *b a*, fill[e] up the Pgr. *f a.* Run *e b* through *b*, and continue[e] to it, *f g.* from *h*, draw *h l*, pall.[f] *g a.* and through *b*, draw *i k*, [f] pall *f h*, and continue *e a* to *l.* X is the pgr. requir'd. For, the △ O X R =[g] Z S V. but O =[g] Z. R =[g] V. Therefore the remains[h] X = (S[i] =) D. and the ang. *a b k* is =[k] (*b*[i] =) *c.* *Q. E. F.*

[d] *Prec.*
[e] *Dem. II.*
[f] *P. I.*
[g] *XXX.*
[h] *Ax.* 8.
[i] *Constr.*
[k] *III.*

Problem XVI.

To make a Pgr. [e f g] (*on a side* [hi] *and ang.* [d] *given*) = *to a rectilinear figure given* [a b c.]

DIvide the rectilinear figure into △s, A, B, C; Then[k] make the pgr. E = A, according to the ang. D. and the side H I. Also F = C, and G = D: These 3 will fall between the same parallels, because[l] their angles are =, and by conseq. make one Pgr. =[m] A B C.

[k] *Prec.*
[l] *VI.*
[m] *Ax.*
LXIII.

Problem XVII.

To frame a Square on a line given [a b.]

n *P. VI.*
o *P. III.*
q *Constr.*
r *Radius's.*
s *XVI.*
t *XIII.*
u *X.*
w *VI.*

FRom A raise a perp.[n] AC = [o] AB. Cent. B and C interv. B A. descr. arches crossing at D, joyn DC, DB. *Q. E. F.* For, AB = [q] AC = [r] CD = [r] DB. CD is com. therefore [s] the △s ACB, CBD, are equiangled. Therefore A[q] = △ (∟ = [s]) D, but ∠ ACB is = [t] ABC, and DCB = [t] DBC. each [u] = ½ a ∟s. therefore C and B are ∟, and the opposite sides are pall [w]. *Q. E. D.*

Problem XVIII.

To make a ▭ [d g, q] = *to a right lin'd figure given.*

x *P. XIV.*
y *P. XI.*
z *CXII. Schol.*

MAke [x] the ▭ BC = A. Between the sides of this ▭ find a mid. propor. [y] DG. The square of this is = [z] (BC, =) A. by constr. *Q. E. F.*

Problem

Problem XIX.

To make a □ [ed], = *to a right-lin'd-figure* [a], *and* □ *given* [bc]; *both together.*

MAke the □ EF = [a] A, and place the corner of it, G, againſt the □ BC, ſo that the ∠ EGB, may [b] be ∟, on the Subtend, EF, [a] make the □ ED, (which will be) [c] = (EF [d] =) A + BC. *Q. E. F.*

[a] *Prec.*
[b] *III.*
[c] *CXIII.*
[d] *Conſt.*

I. *Hence appears how one Square may be ſubtracted from another.*

II. *How a great many Squares given, may be reduced to one Square. For the* □ F, *is* [c] = Eq. (+ Iq. =) Dq. (+ Kq. =) Cq. (+ Lq. =) Bq. + Aq.

Problem XX.

To frame a Pentagon upon a line given [A B.]

DIvide A B in the middle, C; raise the perp. C D = A B. Draw out D A to I, so that A I may be = A C. upon the base A B, make the △ A B F each side = D I. Lastly, upon these sides make the △s F G B, A H F, each side = A B. A B G F H is the Pentagon required. For,

Take F K = A B. and on the cent. A. interv. A C, describe the ○ I C L. The line D I is
[a] *By constr.* [a] = F A, and if you take away the [a] ='s F K,
[b] *Ax.* 8. I L, there [b] remains K A = L D. Now D C
[c] *XLIX*; is a *tangent* [c] to the ○. Therefore [d], I D.
constr. D C :: D C . D L. that is, F A. A B :: A B . A K.
[d] *CXXII.* Wherefore [e] the △ A B F is :: A B K. by con-
[e] *LXXIV.* seq. [f] the ∠ $\zeta = \beta$. and $\varkappa = (\zeta \times \eta$ [g] $=)$
[f] *Def.* 45. ε. Therefore the side B K = (B A =) K F.
[g] *XII.* Therefore the ∠ $\beta =$ [h] η. *viz.* $\zeta = \beta = \eta$.
[h] *XIII.* But the external ∠ $\varkappa$ [i] $= \beta \times \eta$, therefore $\varkappa$,
[i] *IX.* (or ε, or $\zeta \times \eta$) are each double to β. But ε,
[k] *X.* $\zeta \times \eta$, β, together, are [k] = 2 ∟, therefore $\beta = \frac{1}{5}$ 2 ∟ and $\varkappa = \frac{2}{5}$ 2 ∟ (being double to β)
[l] *I.* therefore $\lambda = 1\frac{2}{5}$ 2 ∟. but λ [m] = G [m] = H.
[m] *XVI.* Again, the ∠s F G B, F K B, F H A, are [l] =. (the side B K being prov'd = B A) therefore the ∠s θ, η, δ, α, γ, and β, are all = to one another, and by conseq. = $\frac{1}{5}$ 2 ∟. Therefore $\delta\varepsilon$, $\zeta\eta\theta$, $\alpha\beta\gamma$, are each = $\frac{3}{5}$

 2 ∟.

2 ∟. as H x G have already been prov'd. So that all the 5 ang.'s, are [n] =. And the 5 sides are = by construction. Therefore, *Q. E. F.* [n] *Ax.* 10.

[

Scholium.

The line F A *is cut in extreme and middle reason in the point* K; *for* F A. (A B, i. e.) F K : : F K . K A. *as appears in the precedent.*

Problem XXI.

To make a regular Hexagon, on a line given [a b.]

ON A B, [e] make an equal sided △ A B C. as also on A C, and B C. then drawing [f] out A C and B C, make C F and C E [g] = to them, and joyn G F E D. *Q. E. F.* For, the 3 ang.'s H C I, are = [h] 2 ∟. therefore G D is a [i] right line. Further, the ang. L is = [k] C, and L F, L E are = [l] C A, C B, therefore the △ L F E is = [m] C A B, and so of the other △s, and by conseq. the two angles at E, shall be = 2 at A, and F E = A B, *&c.* So that the whole figure is equal sided and equiangled. *Q. E. D.*

[e] *P. XIII.*
[f] *Lem. II.*
[g] *P. III.*
[h] *XVI, with X.*
[i] *II.*
[k] *III.*
[l] *Constr.*
[m] *XVII.*

Problem XXII.

To make a Polygon (on a line given) [a b] like, *and alike placed, to a Polygon given* [g d.]

n P. IV.
o XVII.
p LXXI.
q LXIII.

DIvide D G into △s, and on A B make the ∠ A B H [n] = C D E; and B A H [n] = D C E, therefore [o] the ∠ H is = E, and by conseq. the △ A H B is :: [p] C E D, and so of the rest: Therefore [q] the whole D K is :: G D. *Q. E. F.*

Problem XXIII.

To make a Polygon [c] like, *and alike placed, to a Polyg. given* [A], *and = to another Polyg. given* [B.]

r P. XVI.
s P. XIV.
t P. XI.
u Preced.
w XCI.
x Constr. bec. D E, H I, E K, are ÷.
y Ax. 10. constr.
z Ax. 10.

ON the line D E, make [r] a Pgr. [s] = A, and on E L, the pgr. Z = [s] B. Take H I a mid. [t] propor. between D E, E K, and make on it the Polyg. C [u] :: A. This C is = B. For, $\frac{A}{C} = {}^{w}\left(\frac{DE}{EK}\ \textit{twice}.\ \text{i.e.}^{x}\ \frac{DE}{EK} = {}^{w}\frac{S}{Z}\ \text{i.e.}^{y}\right)\frac{A}{B}$ Therefore C [z] = B. *Q. E. D.*

Problem

Problem XXIV.

To find the center of an Arch (or Circle) given [a b c] *(or, to draw a ○ through 3 points given.)*

TAke 3 points in the arch. *viz.* A, B, C. joyn A B, B C. divide these in [c] the mid. [a] D and F, by [b] perp's, crossing in E, which is the Center. For each perp. is a [c] Diam. and by [d] conseq. the Center must be where they cross.

[a] *P. VII.*
[b] *P. VI.*
[c] *XXXVII.*
[d] *Def.* 22.

Problem XXV.

To divide an Arch [a c] *in the middle.*

HAving found the Cent. [e] D, draw the Radius A D C, and divide the ∠ D in the [f] mid. by the line D B; therefore the arch A B is [g] = BC. *Q. E. F.*

[e] *Prec.*
[f] *V.*
[g] *XCVI.*

Problem

Problem XXVI.

To draw to Tangent [a d] *from a point given* [a.]

h *P. XXIV.* DRaw a line from A to the Cent. h B, on AB; descr. a Semicirc. cutting the ○ in D; joyn AD, BB. AD, is the Tangent. For the
i *LVI.* ∠ ADB is ∟ i; therefore k *&c. Q. E. F.*
k *XLIX.*

Problem XXVII.

To cut off a Segment [a b] *from a* ○ *given, that may receive an* ∠ = *to an ang. given,* viz. ACB = E.

l *Prec.* DRaw a l Tangent FG, touching in B, make
m *P. IV.* the ∠ FBA = m E. Therefore n the
n *LX.* ∠ ACB (in the opposite Segment) is =
o *Const.* (FBA. o =) E. So that ADCB is the Segment required.

Problem

Problem XXVIII.

To make a △ in a ○, :: a △ given [def.]

CUt [p] off, by AB a Segment receiving an ang. ACB = to the ∠ given, F: Then make BAC [q] = D, joyn BC, the ∠ BCA is = [r] F. Therefore E [s] = B, and by conseq. △ABC :: [t] DEF *Q.E.F.*

[p] *Prec.*
[q] *P.IV.*
[r] *LIV.*
[s] *XXII.*
[t] *LXXI.*

Problem XXIX.

To make a △ about a ○ = a △ given [def.]

MAke the ∠ at the cent. BGA [u] = EDH, and CGA [u] = EFI; and at the ends of the 3 Radius's A, B, C, draw [w] perps, meeting in K, L, N. *Q.E.F.* For, in the *Trapeze*, KBGA, the 4 ∠s are [x] = 4 ∟, (being by a diagonal divisible into 2 △s.) but B and A are [y] ∟; therefore K, G = 2 ∟. Now G = [y] EDH, therefore K = [z] EDF. So also M is = EFD, therefore [a] L = E; and by conseq. [b] the △s are like. *Q.E.F.*

[u] *P.IV.*
[w] *P.VI.*
[x] *X.*
[y] *Constr.*
[z] *I.*
[a] *XXII.*
[b] *LXXI.*

Problem

Problem XXX.

To inscribe a ○ *in a* △ *given* [a b c.]

[c] *P. V.* DIvide the ∠s A and C in the mid.[c] by lines crossing in D. From D, draw perps. to the sides of the △, or D E, D F, D G; cent. D, interv. D F, descr. the ○. *Q. E. F.* For, In the △s A E D, A D F, the ∠s at A are
[d] *Constr.* [d] =: Also A F D, A E D are [e] =, being
[e] *Ax. 2.* ⊥ [d], and the side A D is com. therefore [f] the
[f] *XXIII.* side D E is = D F. = (for the same reason) D G. But D F is a Radius [d], therefore also
[g] *XLVII.* must be D E and D G: and by conseq. [g] the sides B A, B C touch the ○, as A C does, by constr. *Q. E. F.*

Problem XXXI.

To conscribe a ○ *about a* △.

DRaw a Circle through 3 points, by P. XXIV.

Problem

Problem XXXII.

To inſcribe a □ in a ○.

DIvide the Diam. AC in the middle [h], by the [i] perp. BD; joyn B, C, D, A, which is, he □ ſought. For the 4 ſides are [k] =, ſub-nding [l] = ∠s (at the Cent.) between = des [m], and the 4 ∠s ABCD are ∟ [n]; for C is a Diam [o]. *Q. E. D.*

[h] *P. VII.*
[i] *P. VI.*
[k] *XVII.*
[l] *Ax. 2.*
[m] *Radius's.*
[n] *LVI.*
[o] *XXXVII.*

Problem XXXIII.

To conſcribe a ○ about a given □.

DRaw Diams. croſſing perpendicularly [p] at the Cent. I, and at their ends, E, F, G, H, raw 4 [q] perps. meeting in A, B, C, D, which the □ ſought. For, ∠BEG [q] = EGD; herefore BA is [r] pall. CD, in like manner C is pall. AD, and AD pall. EG, and by nſeq. the ∠BAH is (= [ſ] BEI) ∟ [t], and the de AD [u] = EG = [w] FH [u] = BA, &c. thus ll the ſides will be prov'd =, and all the angs. ∟. *Q. E. D.*

[p] *P. VII, and VI.*
[q] *P. VI.*
Ax. 2. con.
[r] *VI.*
[ſ] *IV.*
[t] *Conſtr.*
[u] *XXX.*
[w] *Diams.*

Problem

Problem XXXIV.

To inscribe a ○ in a given □.

x *P. VII.* DIvide the sides AB, BC, in the x mid. by
and VI. by perps. crossing at I. Cent. I, interv.
y *Constr.* IE, desc. a ○. *Q. E. F.* For, ∠ BEG is
z *IV.* ∟ z, = EGD; thus the ∠ FHD, is ∟ also
Therefore IH is (pall. a, and = b EA, =
a *VI.* EB) = a b FI = (by the same process) I
b *XXX.* = IE. But IE is a Radius y, therefore als
must all the rest be, (being = to IE) by con
seq. the ○ touches all the sides of the □
Q. E. D.

Problem XXXV.

To conscribe a ⊙ about a regular Pentagon.

c *P. V.* DIvide the ∠s A and B in the mid. c by line
crossing in F. Cent. F, interv. FB, desc
a ○. *Q. E. F.* For, the ∠ FAB is =
d *Ax: 7.* d FBA. Therefore FB = e FA, = h FC
Constr. because in the △s HG, the side AB = f BC
e *XII.* BF com. ∠ FBA g = FBC. Also, F
f *Suppos.* is h = FD, For, in the △s GI, BC f =
g *Constr.* BD; FC, com. ∠ FCB = FCD. (Fo
h *XVII.* FB is proved = FC; therefore ∠ FB
=

= [i] FCB; therefore FCB is ½ an ang. of the Pent. and by conseq. so is FCD.) Thus all the lines FC, FD, &c. will be proved = FB, which is a Radius [g], therefore also are they; and conseq. the ○ passes by all the angs. of the Pentag. *Q. E. D.* [i] *XIII.*

Problem XXXVI.

To inscribe a ○ in a Pentagon.

CEnt. F. [k] interv. FM (*viz.* a perp. [l] from the cent, to the side) descr. a ○. *Q. E. F.* For, the ∠s at E are [k] =; and (perps. being [l] drawn to all the sides) to ∠ FME = FNE, and the side FE, com. [m] therefore FM is = FM: thus all the perps. will be proved = to FM, which is a Radius [n]; therefore also are they all: and by consequence the ○ touches all the sides of the Polygon. *Q. E. D.*

[k] *Preced.*
[l] *P. VI.*
[m] *XXIV.*
[n] *Constr.*

Problem XXXVII.

To inscribe a (regular) Pentagon in a ○ given.

MAke a reg. Pentag. [o] EB, whose cent. F being found, lay it upon the cent. of the ○; draw the Radius's FH, and through the angs.

[o] *P. XX.*
[p] *P. XXXV.*

angs. of the Pentag. joyn the points G, L, K, &c. *Q. E. F.* For, $\frac{GF}{HF}$ (Radius)[q] $=$ $\frac{EF}{DF}$ (bec. EF [r] $=$ DF.) therefore GH is[s] pall. to ED, and by conseq.[t] $\frac{HF}{DF} = \frac{GH}{ED} =$ (by the same reason) $\frac{HI}{DC}$ &c. But ED $=$ DC, therefore GH $=$ HI, &c. Lastly, upon account of the pall. lines, all the angs. are [u] $=$ respectively, and (by consequence) between themselves; because all the ∠s in the given Pentagon are [w] $=$. *Q. E. D.*

[q] *Ax.* 10.
[r] *Prov'd in P. XXXV.*
[s] *LXXVIII*
[t] *LXIX.*
[u] *IV.*
[w] *Suppos.*

Problem XXVIII.

To inscribe a Hexagon in a ○.

AT the interval of the Radius FA, cut off the arch AB, draw the Rad. FB, therefore [f] is an equal [x] sided △. Then at the interv. of the Rad. FB cut off the arch B, C, &c. therefore Z is equilat. △, in like manner X, &c. therefore DA is a right [y] line, and by conseq. a Diam. But the other half ○, DEA, will admit 3 △s $=$ to the former; and by conseq. the whole ○ admits a regular [z] Hexagon. *Q. E. D.*

[x] *Constr. Rad.*
[y] *X, & II.*
[z] *XXI.*

Problem

Problem XXXIX.

A Polygon in a ○ [b c d] *being given, to inscribe a* like *Polygon* [e f g] *in a* ○ *of any different size.*

UPon the center of the given ○, A, describe the other ○, E F G; draw the Radius's through the ∠s of the Polyg. given, B, C, D, &c. joyn the ends of these Radius's with the right lines E F, F G, &c. The Polyg. E F G will be like the Polyg. B C D. For the △ A B C is :: [o] A E F. [o] *LXX.*

Because the 3 ∠s at A, B, and C, together, are [p] = (2 ∟ [p] =) A, E and F together; therefore taking away from these = summs, the common A, there remains B C = [q] E F. But, B = [r] C, and E [r] = F, therefore B and E, (being like parts, *i. e.* halfs, of = summs) are = [s]; therefore the line E F is [t] pall. to B C. [p] *X.* [q] *Ax.* 10 [r] *XIII.* [s] *LXIII.* [t] *VI.*

In like manner, all the other △'s will be proved *like*; therefore the whole [u] Polygons are like. *Q. E. F.* [u] *LXVII.*

End of the Problems.

If the Reader consider the Difficulty of Printing Books of this nature without faults, and the greater trouble he would be at if they were not faithfully Corrected; 'tis to be hop'd he will pardon this large Catalogue of

ERRATA.

IN *the Advertisement, line* 3. *read* Principles of. *Def.* 6. *l.* 8. *r.* Angles. *Def.* 30. *l.* 2. *r.* [ac, bc.] *marg. add* Fig. 16. *Def.* 31. *marg. dele* Fig. 16. *Def.* 35, 2, *Marg. r.* Fig. 20. *Def.* 41. *Marg. r.* Fig. 37. *Def.* 43. *marg. r.* Fig. 38. *Def.* 51. *l.* 3. *after* angle, *r.* B A E. A, E, being the points where, &c. *Ax.* 3. *l.* 3. *for* [b] *r.* [bd.] *at the end, r.* [bc.] *marg. add* Fig. 6. *Ax.* 12. *l.* 4. *for* A to B. *r.* C to A. *explic. of the Notes. l. last. r.* Hyp. Sup. *add* Q. E. D. which was to be demonstrated. Q. E. A. which is absurd. Q. E. F. which was to be done. *Theor.* I. *l.* 1. *r.* Perp. *Theor.* II. *l.* 4. *r.* ABC. ABE. *Theor.* III. *l.* 3. *for* ^{f} f. *r.* ^{e} e. *l.* 4. *r.* C is ^{f}= to D. *marg. r.* ^{f} *Ax.* 8. *Theor.* 4. *dele* ^{k}. *Theor.* 16. *l.* 3. *r.* BC =. *Theor.* 24. *title l.* 2. *after* are =. *r.* [abc. def.] *p.* 12. *l. last. r.* ^{m} be the. *p.* 13. *l.* 1. *after* therefore, *r.* in the △ Bγ C. *Theor.* 41. *title l.* 1. *for* c d. *r.* a d. *Tb.* 51. *l. last. for* F. *r.* B. *p.* 48. *l.* 4. *for* DGH. *r.* CDG. *Tb.* 55. *l.* 3. *after* B=A. *r.* by 54. *Tb.* 56. *l.* 4. *for* ∟ *r.* 2 ∟. *Tb.* 59. *title l.* 3. *after* Circumference. *r.* [a bc.] *Tb.* 60. *marg. for* ^{h} *L. r.* ^{h} I. *and for* ^{i} I. *r.* ^{i} 62. *Tb.* 63. *l. last. for* C 6. *r.* D 6. *Tb.* 69. *l. last. for* F + G. G. *r.* F + E. (E. i.e.) G. *Tb.* 72. *marg. after* Hyp. *r.* (*viz.* D + C. B + A :: δζ . δε. and D + C. E :: δ. ζ ε ζ) *l.* 3. *r.* C^{d} or. *Tb.* 74. *marg. r.* ^{r} 70. *Tb.* 75. *l.* 2. *for* (δζ ^{r} or) A, δε. *r.* (δε ^{r} or) A δζ. *marg. r.* ^{w} 4. *Tb.* 77. *l.* 1. *for* IH. *r.* FH. *Tb.* 80. *l.* 1. *for* CD. *r.* CBD. *marg. r.* ^{m}. 78. alterned. *Tb*: 81. *marg. for* ^{p} 72. *r.* ^{p}. *Ax.* 7. & 30. *Tb.* 84. *title l.* 1. *after* [s, o] *r.* (i. e. α S β ⊙.) *Tb.* 89. *marg. r.* ^{k} Def. 45.

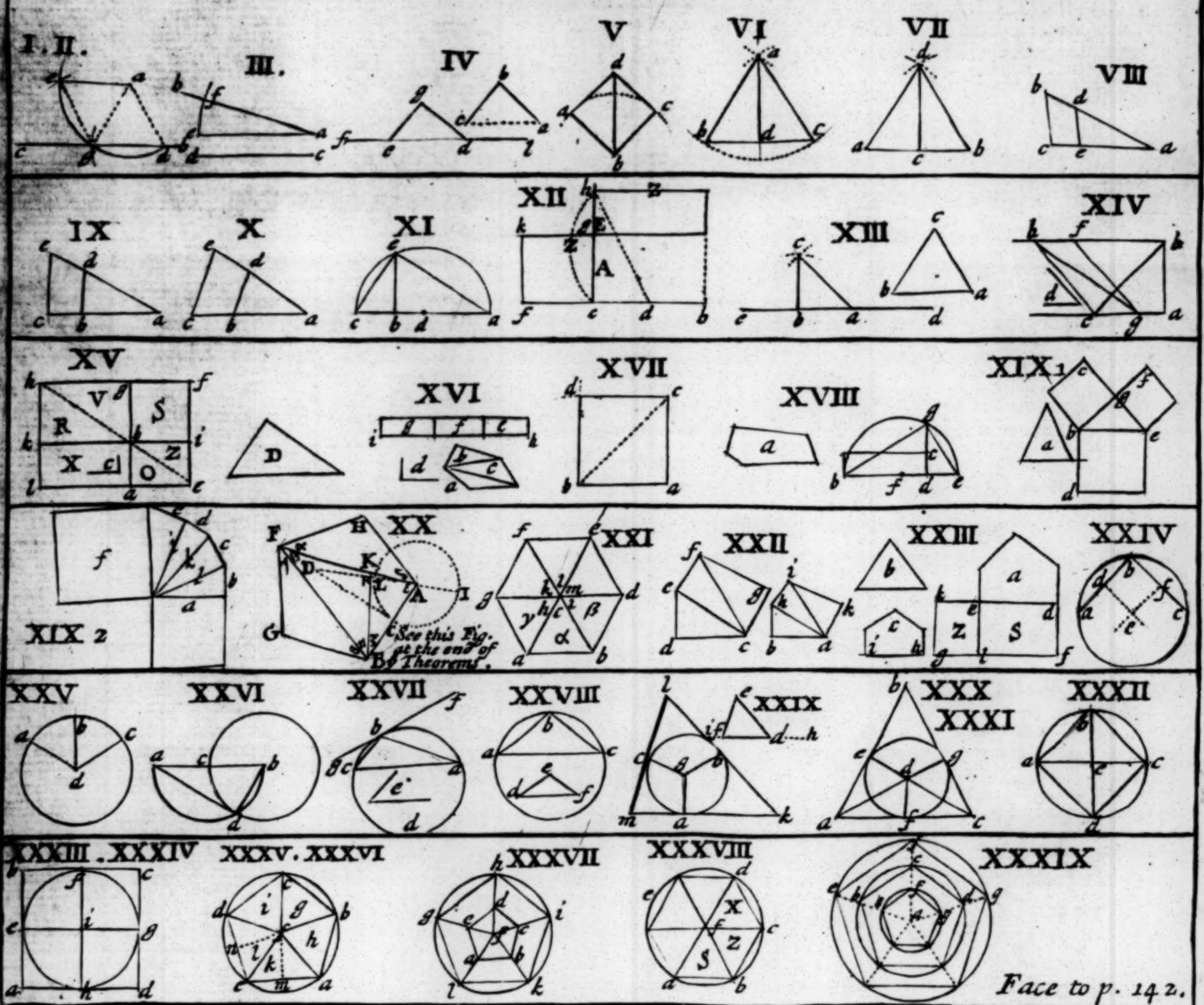
p. 72

ERRATA.

p. 72. *l. last but* 4. *for* F I. *r.* H. I. *last but* 2. *r.* HFI. *Tb.* 92. *l.* 8. *r.* ○ it self; which we conceive to be only a regular Polygon of infinite sides, and by, &c. *marg. after* Probl. *r.* 39. *Tb.* 93. *title. r.* 4 ∟. §. 3. *l.* 2. *r.* each side against the other, so drawn out makes 2 Angles ^{e}. *l.* 4. *after* outward, *r.* at the sides §. 4. *l.* 1. *r.* Angles, at the sides. *Tb.* 94. *l. last but* 4. *dele* 8. *Tb.* 96. *l. last. r.* Arch B D. *Tb.* 97. *l.* 4. *r.* N β. *l.* 7. *r.* O H β. *l. last but* 2. *r.* G Z β. *Tb.* 102. *title l.* 2. *r.* equal in compass. *P.* 80. *l.* 8. *r.* A E or Æ. *l.* 12. *add,* Note also that Z stands for Zumma, or Summa, *i. e.* the whole line; and A, E, most commonly for the parts of it; A the greater, and E the less. *Tb.* 112. *l.* 1. *for* T F. *r.* I F. *marg. r.* ^{d}. Def. 45. *Tb.* 115. *title l.* 1. *after* Triangles, *r.* [A B C.] *Tb.* 122. *after* therefore, *r.* the ∟. *p.* 94. *l.* 2. *r.* {DC {DA. *p.* 96. *l. last. r.* Def. 55. *Tb.* 133. *l.* 4. 8, &c. for 60. *r.* 50. *Tb.* 142. *l.* 4. *for* BE *r.* A E. *l. last but one, for* BAC. *r.* DAC. *Tb.* 144. *marg. r.* ^{k} Def. 78. *and* ^{p} Def. 45. with Ax. 10. *Tb.* 146 *l.* 2. *for* K F. D A. *r.* D C. P A. *p.* 105. *l.* 3. *after* Parallels. *r.* H A. L C. *l.* 4. *after* S Z. *r.* viz. P D. R G. *l.* 5. *for* = prov'd. *r.* prov'd =. *l. last but one. r.* G C. (= E C.) *l. last. r.* A H (= S.). X. *p.* 107. *l. last r.* which $\frac{S}{X}$ thrice = $\frac{AE}{ED}$ *thrice*; *by* 73 ^{u} Q. E. D. For $\frac{S}{X}$ once is = $\frac{AE}{ED}$ once; as hath been prvo'd before. *p.* 109. *l.* 2 *dele* be. *Tb.* 156. *l.* 3. *r.* are in a. *p.* 112. *l. last. for* Q.E.D. *r.* Q.E.A. *Tb.* 158. *title. l.* 3. *for* E Z. *r.* E F. *p.* 115. *l. last but one. for* side. *r.* sphere. *Tb.* 164. *l. last but* 2. *r.* But the Sph. *p.* 117. *l.* 8. for 4 □. *r.* 3 □. *marg. dele* Fig 164. *p.* 120. *l.* 2. *r.* propose. *Prob.* 6. *l* 8. *r.* B C joyn. *l.* 9. *for* time. *r.* line. *p.* 7. *l. last but one. for* D. *r.*

C.

PROBLEMS.



Face to p. 142.

ERRATA.

C. *p.* 9. *l. last.* *r.* for AB. *p.* 11 *title line* 1. *for* bc. *r.* be. *Tex. l.* 2. *after* interv. *r.* dc. *or* da. *p.* 12.) *l. last but* 3. *r. it thus.* Zq = (BC F + CFq *i.e.* q ZA + Aq *p.* 13. *l.* 4. *for* BC. *r.* BE. *p.* 15. *l.* 1. *r.* the ∠ b, being = d c. *p.* 17. *l.* 6. *for* = △ (L. *r.* = (∟. *p.* 18. *title. for* ▭. *r.* □. *after* given. *r,* A. *p.* 19. *l.* 4. *for* EF. *r.* EB. *p.* 20. *for* ζ × *n.* *r.* ζ + *n* *in several places. pag.* 131. *l.* 1 *r.* H and G. *pr.* 23. *l. last but one. for* EK twice. *r.* IH twice. *p.* 26. *l* 3. *r.* AB, BD, AD. *p.* 28. *l.* 4. *after* F. *r.* and BAC = EDF. (conster) *marg. r.* r Sup. with LIV. *p.* 29. *title. for* = *r.* :: *p.* 30. *l.* 3. *for* or, *r.* as. *p.* 32. *l. last. for* AC. *r.* BD. *p.* 33. *title. for* ○ & ▭. *r.* □ & ○. *p.* 36. *l.* 1. *dele* k. *l.* 4. *r.* the ∠. *l.* 6. *r.*) FN *is.* *Pr.* 37. *l.* 2 after found. *r.* (as in Prob. 35.) *l.* 3. *for* and. *r.* &c. *marg. r* f 68. *pa.* 140. *l.* 12. *r.* Pr. 38. *l.* 3. *for* f. *r.* S. *marg. r.* z. Pr. 21.

FINIS.

Problem XXXIX.

'Polygon in a ○ [bcd] being given, to inscribe a like *Polygon* [e f g] *in a ○ of any different size.*

[Pon the center of the given ○, A, describe the other ○, E F G; draw the Radius's rough the ∠s of the Polyg. given, B, C, D, *c.* joyn the ends of these Radius's with the ght lines E F, F G, *&c.* The Polyg. E F G ill be like the Polyg. B C D. For the △ A B C ::[o] A E F. o *LXX.*

Because the 3 ∠s at A, B, and C, together, e [p]=(2 ∟ [p] =) A, E and F together; there- re taking away from these |= summs, the ›mmon B, there remains B C = [q] E F. But, =[r] G, and E [r] = F, therefore B and E, ›eing like parts, *i. e.* halfs, of = summs) e =[s]; therefore the line E F is [t] pa ll. to C. p *X.* q *Ax.* 10. r *XIII.* s *LXIII.* t *VI.*

In like manner, all the other △'s will be oved *like*; therefore the whole [u] Polygons e like. *Q, E. F.* u *LXVII.*

End of the Problems.

ERRATA.

C. p. 9. *l. last.* *r.* for AB. *p.* 11 *title l*[...]
for bc. *r.* b e. *Tex. l.* 2. *after* interv. *r.* d c. *or* d[...]
12.) *l. last but* 3. *r. it thus.* Zq=(BC F+C F[...]
q ZA + Aq *p.* 13. *l.* 4. *for* BC. *r.* BE. *p.* [...]
1. *r.* the ∠ b, being = d c. *p.* 17. *l.* 6. *for* = ∠[...]
r. = (∟. *p.* 18. *title. for* □. *r.* □. *after* give[...]
A. *p.* 19. *l.* 4. *for* EF. *r.* EB. *p.* 20. *for* ζ × η. *r.*[...]
in several places. pag. 131. *l.* 1 *r.* H and G. *pr.* 23.[...]
but one. for EK twice. *r.* IH twice. *p.* 26. *l*[...]
AB, BD, AD. *p.* 28. *l.* 4. *after* F. *r.* and BA[...]
EDF. (conster) *marg. r.* r Sup. with LIV. *p*[...]
title. for = *r.* :: *p.* 30. *l.* 3. *for* or, *r.* as. *p.* [...]
last. for AC. *r.* BD. *p.* 33. *title. for* ○ & □[...]
□ & ○. *p.* 36. *l.* 1. *dele* k. *l.* 4. *r.* the ∠. *l.*[...]
FN *is.* *Pr.* 37. *l.* 2. after found. *r.* (as in Prob.[...]
l. 3. *for* and. *r.* &c. *marg. r* f 68. *pa.* 140. *l.* 12. *r.* Pr[...]
l. 3. *for* f. *r.* S. *marg. r.* z. Pr. 21.

FINIS.

Problem XXXIX.

A Polygon in a ⊙ [bcd] being given, to inscribe a like Polygon [efg] in a ⊙ of any different size.

UPon the center of the given ⊙, A, describe the other ⊙, EFG; draw the Radius's through the ∠s of the Polyg. given, B, C, D, &c. joyn the ends of these Radius's with the right lines EF, FG, &c. The Polyg. EFG will be like the Polyg. BCD. For the △ ABC is ::[o] AEF. [o] *LXX.*

Because the 3 ∠s at A, B, and C, together, are [p] = (2 ∟ [p] =) A, E and F together; therefore taking away from these = summs, the common B, there remains BC = [q] EF. But, B = [r] C, and E [r] = F, therefore B and E, (being like parts, *i. e.* halfs, of = summs) are = [s]; therefore the line EF is [t] pall. to BC. [p] *X.* [q] *Ax.* 10. [r] *XIII.* [s] *LXIII.* [t] *VI.*

In like manner, all the other △'s will be proved *like*; therefore the whole [u] Polygons are like. *Q. E. F.* [u] *LXVII.*

End of the Problems.

To be Corrected and Added.

IN the Advertisement, l. 3. r. *Principles of:* Def. 6. l. 8. r. *Angle S:* l. 9. after *are equal*, add, 3. *The measure of this inclination is (either an arch of a Circle*, BC, *of which the ang.* [a] *is the Cent. or else) a strait line* [gh]; *which being lengthned or shortned (the points* g, h, *remaining still at the same distances from the ang.* a) *will make the inclination of the lines*, ab, ac, *more or less than it is; but if the line* gh, *remains the same, and in the same place, the inclination will be the same. And by conseq. if* IK *be of the same length with* GH, *and the points* IK *at the same distances from the ang.* D; *as* GH *are respectively from* a; *then* DI, DK, *have the same inclination with* AG, AH. Def. 30. marg. add *Fig. XVI:* l. 2. r. [ac, bc.] Def. 31. marg. dele *Fig. XVI.* Def. 46. at the end, add, *And not both Antecedents, and both Consequents in either one of the figures.* Ax. 3. marg. add *Fig. VI.* at the end, r. [bc]. Explication of the notes, at the end, *Q. E. D.*

Q. E. D. which was to be demonſtrated. Q. E. F. *which was to be done.* Pa. 1. l. laſt but 4, r. *Perp.* p. 2. l. 9. r. ABE. p. 8. l. laſt but 5. r. BC = EF: p. 15. to Theor. XXIX. add, *Note, the deſign of this and the* XXVII. *is not to ſhew what lines are pall. but to prove that* = △*s, upon* = *baſes, are of the ſame height.* (Def. 47.) *becauſe all perps. between the ſame palls. are* =, *for ſuch perps. making* (∟*; that is*) = ∠*s, will be pall. themſelves* (VI.) *and by conſeq.* = (XXX.) p. 72. l. laſt but 4. for $\frac{CD}{FI}$ r. $\frac{CD}{HI}$.

K 2 INDEX.

INDEX

With reference to *EUCLID*.

ANGLES.

Bodies, or Solids.

Pyram.

CIRCLES.

LINES.

If

If

Parallelograms, and figures of four ſides.

=

P L A N E S.

Polygons: Figs. of ſeveral ſides.

Pro-

Proportionals.

Rectangles, or the power of lines.

Tri-

Triangles.

△s

FINIS.